

AD-A205 698

THE FILE COPY

0

TACTICAL WEAPON  
**GACIAC**  
GUIDANCE & CONTROL  
INFORMATION ANALYSIS CENTER

GACIAC SR-88-18

## Analytical Modeling of Warfare

Published by GACIAC  
IIT Research Institute  
10 West 35th Street  
Chicago, Illinois 60616-3799

DoD Technical Sponsor:  
U.S. Army Missile Command  
Redstone Arsenal 35898

SDTIC  
ELECTE  
16 MAR 1989  
E



Approved for public release:  
Distribution unlimited

December 1988

30 105

## PREFACE

**Special Report.** This special report is a reprint of lecture notes (with some editing) presented by Dr. Bruce W. Fowler of the Army Missile Command, at a Short Course on Modeling, Simulation and Gaming of Warfare on 23 August 1988 at Georgia Institute of Technology in Atlanta, Georgia. The format consists essentially of a series of viewgraphs with remarks on each line of the viewgraph or a description of the graph depicted. There is sufficient detail that the report represents a tutorial introduction to games and simulations. A large number of parametric plots are included that could serve as a model for assessing the performance of selected weapon systems.

The report is divided into three parts: Definitions and Background; Examples of Games; and, Combat Models. The first part distinguishes between models and simulations. Different types and classifications of simulations are described and interpreted in terms of resolution and detail. The second part comments on the MinMax theorem, the Col. Blotts game, and the Ancient Empires game. The conclusions that may be obtained from games are outlined. Less than a third of the report is devoted to these two parts.

Much of the report is contained in the third part, "Combat Models". The factors of war, processes of combat, and the Lanchester Laws are introduced. Background on both the Lanchester Linear Law and Lanchester Quadratic Law is presented. The author mentions his derivation of a mixture of both Lanchester Laws in a particular combat scenario. Extensive parametric data are presented on short battles in history. About half of this part, or one third of the overall report, concerns the application of a transport theory of combat based upon different approaches to attrition and the consequences for the use of smart munitions in combat. This discussion provides a theoretical basis for assessing the application of smart munitions on the modern battlefield.

**Reproduction.** Permission to reproduce any material contained in this document must be requested and approved in writing by the U.S. Army Missile Command, ATTN: AMSMI-RD-SM, Redstone Arsenal, Alabama 35898-5246. This document is only available from GACIAC, IIT Research Institute, 10 West 35th Street, Chicago, Illinois 60616-3799.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

|   |   |   |                                   |
|---|---|---|-----------------------------------|
| 1a. REPORT SECURITY CLASSIFICATION<br>UNCLASSIFIED  |   | 1b. RESTRICTIVE MARKINGS<br>Approved for public release<br>Distribution unlimited                       |                                   |
| 2a. SECURITY CLASSIFICATION AUTHORITY   |   | 3. DISTRIBUTION/AVAILABILITY OF REPORT  |                                   |
| 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE   |   |   |                                   |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S)<br>GACIAC SR-88-18  |   | 5. MONITORING ORGANIZATION REPORT NUMBER(S)   |                                   |
| 6a. NAME OF PERFORMING ORGANIZATION<br>GACIAC<br>IIT Research Institute   | 6b. OFFICE SYMBOL<br>(If applicable)    | 7a. NAME OF MONITORING ORGANIZATION<br>U.S. Army Missile Command,<br>ATTN: AMSMI-RD-SM                  |                                   |
| 6c. ADDRESS (City, State, and ZIP Code)<br>10 West 35th Street<br>Chicago, IL 60616-3799  |   | 7b. ADDRESS (City, State, and ZIP Code)<br>Redstone Arsenal, AL 35898-5246                              |                                   |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION<br>U.S. Army Missile Command  | 8b. OFFICE SYMBOL<br>(If applicable)    | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER<br>DLA900-86-C-0022                                     |                                   |
| 8c. ADDRESS (City, State, and ZIP Code)<br>ATTN: AMSMI-RD-SM<br>Redstone Arsenal, AL 35898-5246   |   | 10. SOURCE OF FUNDING NUMBERS   |                                   |
|   |   | PROGRAM<br>ELEMENT NO.<br>65802S  | TASK<br>NO.<br>1.0                |
|   |   | WORK UNIT<br>ACCESSION NO.  |                                   |
| 11. TITLE (Include Security Classification)<br>Analytical Modeling of Warfare   |   |   |                                   |
| 12. PERSONAL AUTHOR(S)<br>Dr. Bruce W. Fowler   |   |   |                                   |
| 13a. TYPE OF REPORT<br>Special Report   | 13b. TIME COVERED<br>FROM 8-88 TO 12-88 | 14. DATE OF REPORT (Year, Month, Day)<br>December 1988  | 15. PAGE COUNT<br>201             |
| 16. SUPPLEMENTARY NOTATION<br>This report is only available from GACIAC. Reproduction is not authorized except by specific permission. 410948 \$50.00   |   |   |                                   |
| 17. COSATI CODES  |   | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)                       |                                   |
| FIELD   | GROUP                                   | SUB-GROUP   |                                   |
| 15  | 06                                      |   |                                   |
| 12  | 05                                      |   |                                   |
|   |   | War Games, Warfare, Models, Simulation, Analytical Study, Mathematical Models, Definitions, Game Theory |                                   |
|   |   | Combat Simulation   |                                   |
| 19. ABSTRACT (Continue on reverse if necessary and identify by block number)<br>The presentation was a Short Course on Modeling, Simulation and Gaming of Warfare presented at Georgia Institute of Technology on August 23, 1988. This presentation is an introduction to the analytical modeling of warfare. The emphasis is on the Lanchester attrition equations. The presentation is divided into three parts: a section on definitions and background, a section with examples on Game Theory, and a final section on Combat Models. The last section is the bulk of the presentation. The middle section on Game Theory is brief mostly because of the technical complexity of the material and the specialized nature of its application in the area of warfare modeling; it is included for completeness. The first section is intended to provide us with a common set of terms and vocabulary. |   |   |                                   |
| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT<br><input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS   |   | 21. ABSTRACT SECURITY CLASSIFICATION<br>UNCLASSIFIED/UNLIMITED  |                                   |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL<br>Chalmer D. George  |   | 22b. TELEPHONE (Include Area Code)<br>(205) 876-3171  | 22c. OFFICE SYMBOL<br>AMSMI-RD-SM |

THIS PAGE IS INTENTIONALLY BLANK

# Analytical Modeling of Warfare

## Modeling, Simulation and Gaming

Dr. Bruce W. Fowler  
U.S. Army Missile Command  
AMSMI-RD-AC  
Redstone Arsenal, Alabama 35898-5246

Presented at a Short Course on  
Modeling, Simulation and Gaming of Warfare  
23 August 1988  
Georgia Institute of Technology  
Atlanta, Georgia

Published by GACIAC  
IIT Research Institute  
10 West 35th Street  
Chicago, Illinois 60616-3799



|                    |                                     |
|--------------------|-------------------------------------|
| Accession For      |                                     |
| NTIS GRA&I         | <input checked="" type="checkbox"/> |
| DTIC TAB           | <input type="checkbox"/>            |
| Unannounced        | <input type="checkbox"/>            |
| Justification      | 150.00                              |
| By _____           |                                     |
| Distribution/      |                                     |
| Availability Codes |                                     |
| Dist               | Avail and/or Special                |
| A-1                | 21                                  |

Copies only from GACIAC  
Reproduction not authorized except  
by specific permission

Approved for public release:  
Distribution unlimited

# TABLE OF CONTENTS

|  | Page |
|--|------|
| Outline .....  | 1    |
| Definitions .....  | 3    |
| Descriptions .....   | 5    |
| Model vs. Simulation .....   | 7    |
| Types of Simulations .....   | 9    |
| Informational Simulations .....  | 11   |
| Event vs. Time Sequencing .....  | 13   |
| Event Sequence Simulation .....  | 15   |
| Event Sequence Simulation .....  | 17   |
| Simulation Characteristics .....   | 19   |
| Scale .....  | 21   |
| Abstraction vs. Detail .....   | 23   |
| Resolution vs. Detail .....  | 25   |
| Abstraction vs. Resolution vs. Detail .....                                | 27   |
| Simulation Classification .....  | 29   |
| Outline .....  | 31   |
| MinMax Theorem .....   | 33   |
| Colonel Blotto Game .....  | 35   |
| Colonel Blotto Payoff Matrix .....   | 37   |
| Games of Timing--Duels .....   | 39   |
| Ancient Empires, Model Characteristics .....                               | 41   |
| Ancient Empires .....  | 43   |
| Ancient Empires .....  | 45   |
| Ancient Empires .....  | 47   |
| Ancient Empires .....  | 49   |
| Outline .....  | 51   |
| Factors of War .....   | 53   |
| Processes of Combat .....  | 55   |
| Weapons are Ominous Tools to be Used Only When There is no Alternative ... | 57   |
| Lanchester Law Assumptions, Linear Law Assumptions .....                   | 59   |
| Lanchester Law Assumptions, Quadratic Law Assumptions .....                | 61   |
| Attrition Law Decision Tree .....  | 63   |
| Lanchester Law Differential Equations .....                                | 65   |
| State Space Solutions, Linear Equations .....                              | 67   |
| State Space Solutions, Quadratic Equations .....                           | 69   |
| State Space Solutions Linear Equations .....                               | 71   |
| Quadratic Lanchester Equation, Force Strength Variation .....              | 73   |
| Quadratic Lanchester Equation, Attrition Rate Variation .....              | 75   |
| Time Solutions, Linear Equations .....                                     | 77   |
| Time Solutions, Quadratic Equations .....                                  | 79   |
| Time Solutions, Mixed Equations .....                                      | 81   |
| Victory Prediction .....   | 83   |
| Short Battles, Initial Force Strength vs. Date .....                       | 85   |
| Short Battles, Final Force Strength vs. Date .....                         | 87   |
| Short Battles, Force Strength Losses vs. Date .....                        | 89   |
| Short Battles, Normed Force Strengths vs. Duration .....                   | 91   |
| Short Battles, Initial Force Strengths vs. Duration .....                  | 93   |
| Short Battles, Final Force Strengths vs. Duration .....                    | 95   |

# TABLE OF CONTENTS (Continued)

|  | Page |
|--|------|
| Short Battles, Loss in Force Strength vs. Duration .....                   | 97   |
| Short Battles, Delta Defender vs. Average Attacker .....                   | 99   |
| Short Battles, Delta Attacker vs. Average Defender .....                   | 101  |
| Short Battles, Delta Force Strength vs. Average Force Strength Product ... | 103  |
| Short Battles, Final vs. Initial Force Strengths Attacker .....            | 105  |
| Short Battles, Final vs. Initial Force Strengths Defender .....            | 107  |
| Short Battles, Attacker vs. Defender Force Ratios .....                    | 109  |
| General Observation About the Lanchester Laws .....                        | 111  |
| Transport Theory of Combat .....   | 113  |
| Two Approaches to Attrition Processes .....                                | 115  |
| Attrition Process, Conventional Form .....                                 | 117  |
| Attrition Process, Smart Munition Form .....                               | 119  |
| Attrition Process .....  | 121  |
| Serial vs. Parallel Process .....  | 123  |
| Serial vs. Parallel Attrition .....  | 125  |
| Target Acquisition Events .....  | 127  |
| Target Acquisition .....   | 129  |
| Hull Defilade Effects .....  | 131  |
| Acquisition Time, Random Glimpse; Hull Defilade Target .....               | 133  |
| Acquisition Time, Random Glimpse; Fully Exposed Target .....               | 135  |
| Probability That Object is in Search Area, Linear Formation .....          | 137  |
| Probability of a Clear LOS .....   | 139  |
| Normal Probability Distribution .....                                      | 141  |
| Target Signal/Noise .....  | 143  |
| Threshold Contrast, Inherent Contrast = 0.2 .....                          | 145  |
| Probability of Detection Fully Exposed, Visual Spectral Band .....         | 147  |
| Probability of Detection Hull Defilade, Visual Spectral Band .....         | 149  |
| Target Engagement Process .....  | 151  |
| Engagement Process, Times and Probabilities .....                          | 153  |
| Target Selection, Target Rejection .....                                   | 155  |
| Bonder's Equation .....  | 157  |
| Attrition Time .....   | 159  |
| Acquisition Factors .....  | 161  |
| Engagement Factors .....   | 163  |
| Expected Times, Conventional Form .....                                    | 165  |
| Normal Probability Distribution, Centered Target Approximation .....       | 167  |
| Square Target Uniform Sigma Approximation, Dispersion Sigma .....          | 169  |
| Horizontal Dispersion .....  | 171  |
| Vertical Dispersion .....  | 173  |
| High Angle Vertical Dispersion .....                                       | 175  |
| Accuracy Effects, Aim Point Center .....                                   | 177  |
| Accuracy Effects, Aim Point Effects .....                                  | 179  |
| Accuracy Effects, Range Effects .....                                      | 181  |
| Probability of Hit .....   | 183  |
| Range Variable Attrition Rate .....  | 185  |
| Force Strength, Fixed Attrition Rates .....                                | 187  |
| Force Strength, Variable Attrition Rates .....                             | 189  |
| Force Strength, Attacker .....   | 191  |
| Force Strength, Defender .....   | 193  |

## OUTLINE

The report is divided into three parts: a section on definitions and background, a section with examples on game theory, and a final section on combat models. The final section is the bulk of the report. The middle section (on game theory) is brief, mostly because of the technical complexity of the material and the specialized nature of its application in the area of warfare modeling; it is included for completeness. The first section is intended to provide a common set of terms and vocabulary.



# OUTLINE

## 1. Definitions and Background

## 2. Examples of Games

## 3. Combat Models

## DEFINITIONS

First of all, we distinguish between a model and a simulation. You will find that this distinction is not always made in the literature, but is useful in drawing a distinction between the conceptual basis for understanding combat and the operational basis for applying this understanding.

To proceed with this, we must first assume that there exists some correspondence between reality and our perceptions of that reality which we refer to as the real world. For the purpose of our discussions here, we shall draw no strong distinction between reality and the real world. We shall take the two as being equivalent.

A model is a conceptual device which represents some aspect of reality (that which is going on around us), in a form which permits that aspect to be understood. An example of this is the simple statement "If there are dark clouds in the sky, it will rain." This is a model. It translates some aspect of reality (actually two--rain and dark clouds) into understandable terms: in this case, a condition (dark clouds) and an effect (rain). Notice that the model here is not causal--we do not imply that dark clouds are the cause of rain, merely a signature which precedes rain.

A simulation, on the other hand, is a tool, made up of one or more models, which is used to predict some aspects of reality. If we use our cloud-rain model by watching the sky, and conclude, whenever we see dark clouds, that rain will follow, we are exercising a simulation. We are making a prediction of some aspect of reality--in this case, rain.

# DEFINITIONS

|            |  |
|------------|--|
| Model      | A mapping of reality into<br>comprehensibility.      |
| Simulation | A prediction of reality using one<br>or more models. |

## DESCRIPTIONS

Some characteristic differences between models and simulations will help express the difference between the two. For example, models may always (indeed, must always) be expressed in informational symbology. This is a fancy way of saying that a model must always be expressed in terms of words or pictures or some such equivalent. These equivalents include mathematics. Although this is dressed up in a lot of fancy terminology, what it fundamentally says is that a model consists only of information. It is this information which is used to build the simulations.

Simulations are built from one or more models. If a simulation does not incorporate a model (or models) of some aspect of reality, then that simulation will not reflect the effects of that aspect. This is a two-fold statement. For example, if a model represents some aspect of reality which is conditional for a prediction (the prediction cannot be made without that aspect being known), then the simulation which does not include that model will either not predict that aspect of reality, or will predict incorrectly at least some of the time. We may consider this in terms of our model in the form: "If dark clouds, then rain." This is the simplest form of a model where there is a precondition (expressed by the If) and a post-condition (or result--expressed by the Then). In a simulation utilizing this model, if the precondition is missing (the simulation does not know whether there are dark clouds or not), then the result (rain) either cannot be predicted, or may be predicted incorrectly.

This may seem like a trivial example, but it points out one of the greatest misuses of simulations. Many managers and study executives get false results in their studies because their analysts haven't asked, "what are the limitations of the models in the simulations being used?"

# DESCRIPTIONS

Models may always be expresses in informational symbology.

Simulations lacking models of an aspect of reality will not reflect the effects of that aspect.

## MODEL vs. SIMULATION

To get back into the vein of our lesson, consider a model that describes the probability of kill of a gunman firing at a target. This is a complex model which has multiple preconditions and a probabilistic result--a result which is not certain. We may use this model to build a simulation of a formal duel between two gunmen. The performance of each gunman is described by the model. The goal of the simulation is to predict the outcome of the duel.

This is a case where the simulation is made up of two models, or rather, the same model used twice. If we designate the two gunmen by the letters A and B, then the aspects of both A and B are preconditions to the model. In one case, gunman A will fire on gunman B (the target in the model), while in the other case the role of the two is reversed.

# Model vs. Simulation

Model: a gunman's performance is described by a probability of kill (hit) as a mathematical function of several variables (e.g. target size, range, light conditions, etc.) supported by underlying assumptions and conditions.

Simulation: two gunmen (A and B) conduct a formal duel. The performance of each is expressed by the above model. What is the outcome of the duel?

## TYPES OF SIMULATIONS

This chart lists the basic types of models used in simulations.

Iconic simulations are those where the model and the simulation are identical. An example of this is a model of an airplane to be placed in a wind tunnel. The informational model of the airplane is expressed in the material form of the simulation--in the vernacular, the "model" airplane itself. (Technically, the "model" airplane is really only a sub-simulation, since a wind tunnel and its operation with the "model" inside are necessary to "simulate" the flight of the airplane).

Analog simulations use double translation models. They work by the logical process: Precondition A is related to precondition C; If C, then D, and result D is related to result B as C is to D.

We can draw a diagram of this as:

A  $\Leftrightarrow$  C  
C  $\Rightarrow$  D  
D  $\Leftrightarrow$  B

The best example of simulations of this type are the parables in the Bible. The third type of simulation is the symbolic simulation. These are the most common from our viewpoint.

The first subtype of a symbolic simulation is the mechanistic simulation. The simulation is material. The simulation mentioned above of a wind tunnel and a "model" airplane is a mechanistic simulation. A slide rule is another example of a mechanistic simulation. The model used in a slide rule is  $\log(A \cdot B) = \log(A) + \log(B)$ .

Multiplication (and division) are represented by the addition (subtraction) of lengths on two rulers which are scaled logarithmically.

Another subtype of symbolic simulation is the informational simulation. The best examples of this are computer code simulations.

The third subtype is the simulation which is mixed, embodying both informational and mechanistic components. Most board games are of this subtype. The board, playing pieces, and dice are mechanistic; the rules are informational.



# Types of Simulations

1. Iconic (Model = Simulation)
2. Analog (Parables)
3. Symbolic
  - a. Mechanistic (Slide Rule)
  - b. Informational (Code)
  - c. Mixed (Board Game)

## INFORMATIONAL SIMULATIONS

There are two fundamental classes of informational simulations: deterministic and stochastic (or probabilistic). Deterministic simulations produce definite results; stochastic simulations produce probabilistic results.

Simulations embodying the Lanchester equations that we will describe here are deterministic simulations. (There are stochastic forms of the Lanchester equations, but we will not discuss them here).

There are two subclasses of stochastic simulations: event sequenced and time sequenced. Stochastic simulations are also called Monte Carlo simulations after Monte Carlo (stochastic) processes.

# Informational Simulations

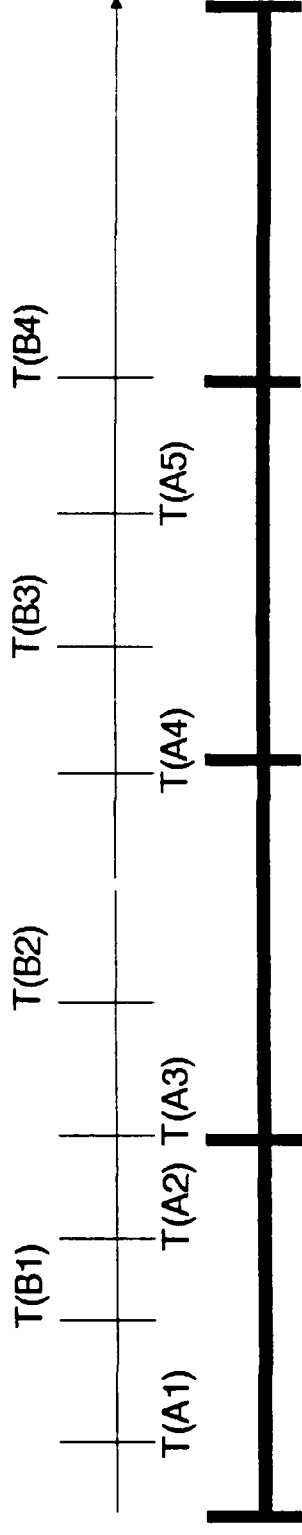
1. Deterministic
2. Stochastic (commonly Monte Carlo)
  - a. Time Sequenced
  - b. Event Sequenced.

### EVENT vs. TIME SEQUENCING

This diagram depicts the equivalence and differences of event and time sequenced simulations. In event sequenced simulations, the simulation simulates events in the time order in which they occur, although time as an independent variable may have been removed. (We will present an example of this later.) Time sequenced models simulate all events that occur in a given time interval, but not necessarily in the time order in which they occur. Care must be taken in time sequenced simulations that spurious results are not introduced by ignoring the rigorous time sequencing of the events.

# Event vs. Time Sequencing

Two processes, A and B, have events  $A_i$  and  $B_j$ , which occur at times  $T(A_i)$  and  $T(B_j)$



Event Sequenced Simulations execute the events in the order of their occurrence:  $T(A1), T(B1), T(A2), T(A3), T(B2), T(A4), T(B3),$

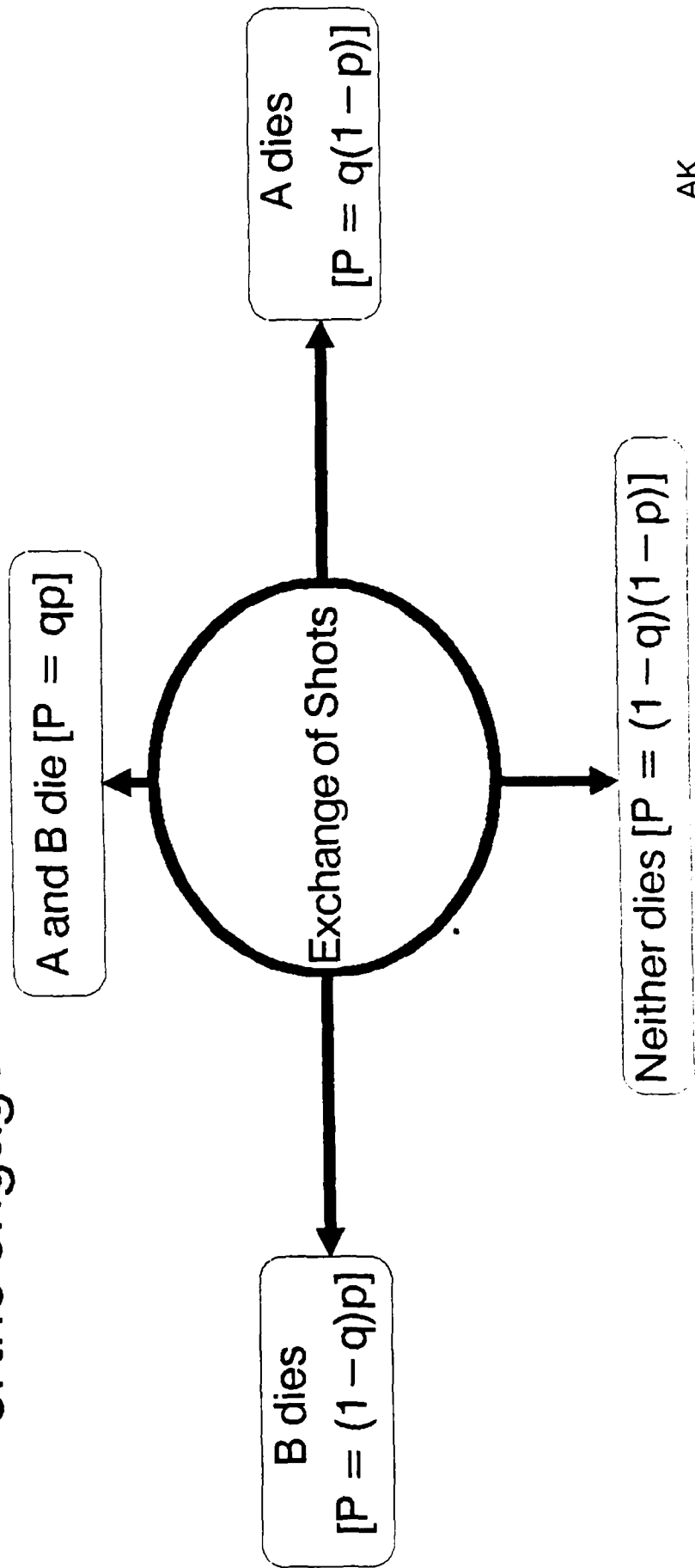
Time Sequenced Simulations execute all events in a time interval as if they were simultaneous:  
 $T(A1), T(A2), T(B1); T(A3), T(A4), T(B2); T(A5),$

### EVENT SEQUENCE SIMULATION

This is a diagram of an event sequenced simulation of our formal duel example. Because the duel is formal, the exchange of shots is always simultaneous (if both gunmen fire--an assumption of the simulation). The simulation produces the probability associated with the four possible outcomes of an exchange of shots: both gunmen killed, one of the two gunmen killed (two possibilities), or neither gunman killed.

# Event Sequence Simulation

Building formal duel simulation: Model variables (participant physical dimensions, 20 pace range, dawn, etc.) gives probability of kill (incapacitation) of  $p$  and  $q$ . The subsimulation of the engagement is:



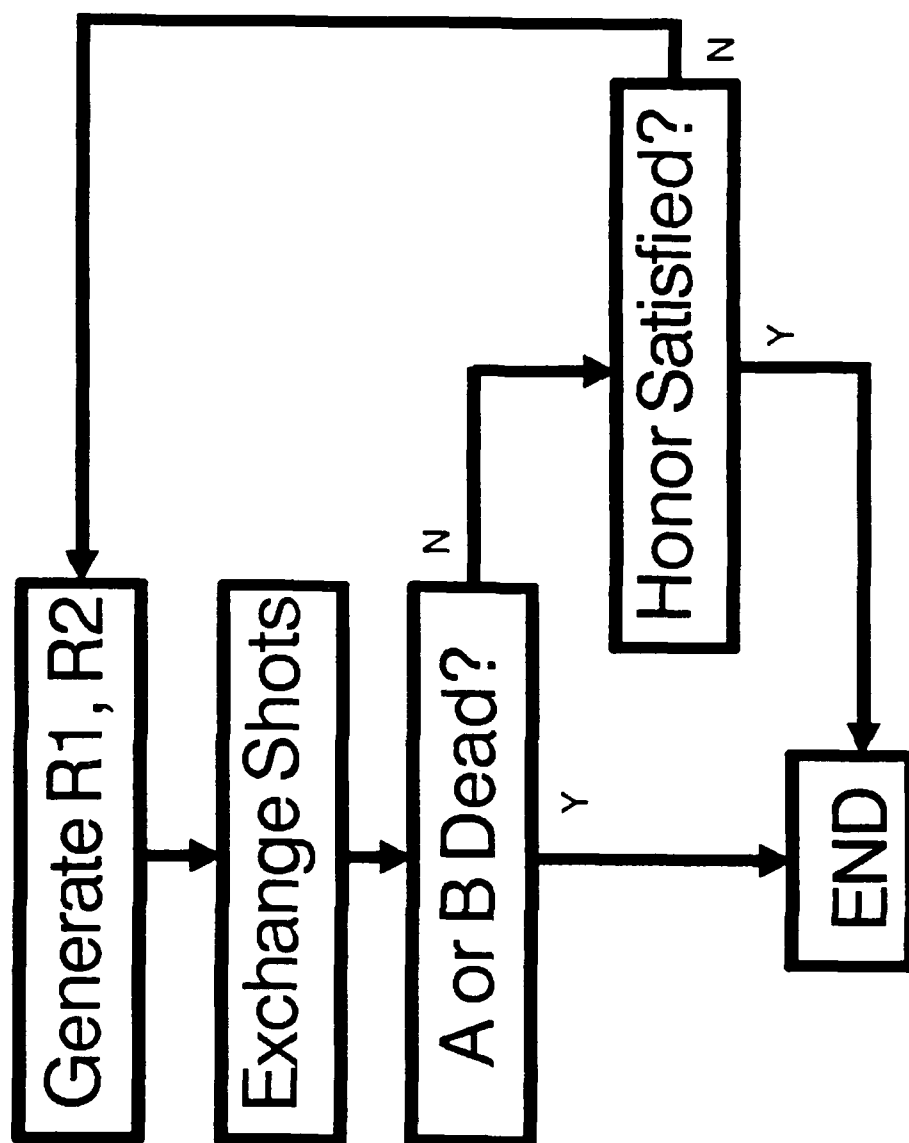
### EVENT SEQUENCE SIMULATION

The rest of the simulation of the event sequenced duel is diagrammed by means of a flow chart. Two random numbers are generated, and the portion of the simulation described above is exercised. A test is made using the random numbers and the probability outcomes of the exchange sequence simulation to determine if one or both of the gunmen are killed; if so, the simulation ends. If neither is killed, then a subsimulation of whether honor has been satisfied is executed. If honor has been satisfied, the simulation ends; if not, then the exchange of shots is repeated with a new pair of random numbers. In principle, the duel simulation can continue until one (or both) of the gunmen is killed.



# Event Sequence Simulation

II. FLOW CHART



### SIMULATION CHARACTERISTICS

We list here some of the principal characteristics of simulations. These are described in the following charts.

# Simulation Characteristics

Scale

Abstraction vs. Detail

Resolution vs. Detail

Spatiotemporal Representation

Outcome Assessment

### SCALE

The scale of a simulation is the size of the simulation. Several aspects are described here. The unit size scale may range from an individual soldier to an entire army. The space scale may range from a square meter to the entire world. The increment of space may vary similarly.

# Scale

Unit Size: Individual troop -> Corps/Army

Space : Square/Cubic Meters (man-to-man) -> Global

Space Increment: Meter -> Kilometers

Time: Subseconds -> years

Time Increment: Subsecond -> months

### ABSTRACTION vs. DETAIL

Any simulation balances abstraction with detail. If we view a simulation as a set of representations of processes, then the detail of the representations may vary within the simulation. Generally, a simulation is designed to expressly provide certain information within constraints.

Processes are important in providing information which will likely have detailed representations, while those which are less important in providing that information likely have abstracted representations. An example of this may be found in a simulation of an infantry battle. The dynamics of combat between the two infantry forces will likely be represented in a detailed manner (e.g., the individual soldier and shots from his weapons), while the effect of artillery in the combat may be represented in an abstract manner (e.g., on the impact points and resulting effects of the shots, but not the placement of the guns themselves).

# Abstraction vs Detail

Process Representation

Mixture of Representations

Detailed Representations of Interesting/Simple/Critical Processes

Abstracted Representation of Uninteresting/Complex/Non-critical Processes

### RESOLUTION vs. DETAIL

Another characteristic of simulations is resolution. This is the degree to which units, dimensions, and time are represented. In our infantry battle simulation example, the representation of the infantry units is highly resolved (individual soldier), while the artillery units may only be resolved at the unit (e.g., battery) level.



# Resolution vs. Detail

Unit Representation

Dimension Representation

Time Representation

#### ABSTRACTION vs. RESOLUTION vs. DETAIL

As you will have noted by now, abstraction, resolution, and detail are related. In general, when the resolution is low, the processes involving that resolution are usually fairly abstract.

# Abstraction vs. Resolution vs. Detail

These three are related: if units are divisions (resolution), then attrition process is complex and the model is simplified (abstraction).

## SIMULATION CLASSIFICATION

Finally, simulations may be classified on the basis of the level of the force that they simulate and the type of interactions between those forces. Force-on-Force simulations represent combat between balanced forces, for example, division on division. These simulations often have fairly low resolution.

Many-on-Many simulations generally have higher resolution than Force-on-Force simulations, but have smaller force sizes simulated. Few-on-Few simulations generally simulate combat between a small number of units on each side (e.g., a company or fewer).

One-on-One simulations, as their name implies, simulate combat between two units only.

One-sided simulations are characterized by one force being highly resolved while the other force is either entirely absent or represented in a very abstract, lowly resolved form. An example of this is a simulation for determining the basic load and rearm rate for artillery.

Engineering simulations are usually either One-on-One or Few-on-Few simulations where all of the units but one particular type are of very low resolution. That one particular unit is simulated with very highly detailed models which facilitate determination of the performance of the unit when the engineering aspects of the weapon are changed. The purpose of these simulations is the design of units or their weapon systems. Training simulations have a high level of abstraction except for those aspects of the unit(s) which are important in teaching those aspects of the unit(s). These simulations are designed to have a relatively high degree of human interaction during the course of the execution of the simulation.

Finally, doctrinal simulations are used to develop combat doctrine. They will have high resolution aspects which reflect the changes in doctrine.

# Simulation Classification

Force – On – Force

Many – On – Many

Few – On – Few

One – On – One

One Sided

Engineering

Training

Doctrinal

## OUTLINE

The second section of this report is concerned with game theory.

# OUTLINE

1. Definitions and Background

2. Examples of Games

3. Combat Models

## MINMAX THEOREM

The fundamental basis of game theory is the MinMax Theorem. If two sides **B** and **R** (for blue and red), interact in some manner (e.g., combat), and they have strategies in that interaction which result in some outcome of that interaction, then the determination of the optimal selection of strategy by one side is described by the MinMax Theorem.

The outcome of each pair of blue-red strategies is assumed to be describable by some mathematical function, called a payoff. The value of the payoff indicates the relative merit of the pair of strategies. If each side has several strategies, then the payoff for all of the strategies may be expressed in a matrix (called the Payoff matrix).

What the MinMax Theorem states is that there exists (at least in principle) a pair of strategies which minimizes the gain for one side and minimizes the loss for the other side, and that the order of determining that optimal pair of strategies does not depend on the order in which the maximization and minimization are performed.

Further, the MinMax Theorem states that the optimal strategy minimizes the losses (maximizes the gains) of the losing side and minimizes the gain (maximizes the losses) of the winning side.

As you might have figured out by now, game theory is of a great deal of interest in doing things like determining strategy but not in simulating combat.



# MinMax Theorem

If two sides B and R have strategies  $i$  and  $j$ , respectively, with payoffs  $P_{ij}$  then:

$$\text{Max}(\text{wrt } i) \text{ Min}(\text{wrt } j) P_{ij} = \text{Min}(\text{wrt } j) \text{ Max}(\text{wrt } i) P_{ij}$$

and there exists an optimal strategy  $i^*$   $j^*$  such that

$$P_{i^*j} \geq P_{i^*j^*}$$

and

$$P_{ij^*} \leq P_{i^*j^*}$$

### COLONEL BLOTTO GAME

An example of a simple (not a lot of strategies) game is the Colonel Blotto game. In this game, Col. Blotto has four units, his enemy has three. There are two objectives to be occupied (e.g., mountain passes). Both sides would like to control both objectives.

The payoff is given by the equation. Simple force strength is assumed to determine battle outcome. If one force outnumbered the enemy at an objective, then that force wins. The payoff is the number of enemy units plus one. The equations are structured from Col. Blotto's point of view.

# Colonel Blotto Game

Col. Blotto commands B. His opponent commands R. They have two objectives. Their initial strengths are:  $B = 4$ ,  $R = 3$ . They must divide their troops between the two objectives.

The Payoffs at each objective are:

$$B > R; P_{ij} = R + 1$$

$$B < R; P_{ij} = -B - 1$$

$$B = R; P_{ij} = 0$$

All payoffs are relative to Col. Blotto's strategy.

### COLONEL BLOTTO PAYOFF MATRIX

The two payoff matrices for the two sides is given here. Col. Blotto's strategies are given by the rows, his opponent's by the columns. We see immediately that although there are apparently five blue and four red strategies (a total of 20 strategy pairs), there are really only three independent blue and two independent red strategies. The bottom row is the maximum values of each column; the sixth column is the minimum values of each row.

If we examine the column of minimums, we see that the maximum of the minimum returns to Blotto is the strategy of sending four units to one objective and no units to the other. The optimal strategy for Red is to send two units to one objective and one unit to the other. From Blotto's standpoint, this ensures that he will at worst break even (zero score) if Red sends all of its units to the objective that he sends no units to. From Red's standpoint, their strategy ensures that Blotto can get no greater return than a score of two, and more importantly from their standpoint, ensures that they control one objective.

Note that if Blotto chooses to select a strategy that minimizes his maximum gain, he opts to send two units to each objective. This is a poor choice, since then Red sends all of its forces to one objective and ensures a net loss for Blotto.

# Colonel Blotto Payoff Matrix

|       | (3,0) | (0,3) | (2,1) | (1,2) | Mins |
|-------|-------|-------|-------|-------|------|
| (4,0) | 4     | 0     | 2     | 1     | 0    |
| (0,4) | 0     | 4     | 1     | 2     | 0    |
| (3,1) | 1     | -1    | 3     | 0     | -1   |
| (1,3) | -1    | 1     | 0     | 3     | -1   |
| (2,2) | -2    | -2    | 2     | 2     | -2   |
| Maxs  | 4     | 4     | 3     | 3     |      |

BC

### GAMES OF TIMING--DUELS

While discussion of stochastic duels is beyond the scope of this report, we do introduce the definitions of three different types of duels. Besides the formal duel which we have already discussed, there are also noisy and silent duels which are typified by the information that the gunmen receive about each other's fire.

# Games of Timing - Duels

- ▶ Noisy Duel - A duelist learns of his opponent's moves when they happen.
- ▶ Silent Duel - A duelist never learns of his opponent's moves at all.
- ▶ Formal Duel - Both opponents fire at the same time.

## ANCIENT EMPIRES

### Model Characteristics

In a game called "Ancient Empires", terrain/space is abstracted as countries. Time is abstracted to two levels: the move and the engagement. (The latter is a sub-set of the move during engagements.) The game is a mixed event/time sequenced simulation where events within a country occur in event sequence during a combat, but may also occur in event sequence across countries, such as during reinforcement.

The logistics aspects of the game are highly abstracted, occurring automatically at the end of each move.

Combat occurs obeying Lanchester Square Law Attrition (discussed in the next section). Each side simultaneously attacks the other. There is no advantage to the defender. (In contemporary terms, only meeting engagements are allowed.) There are three types of units, and allocation of forces for combat is limited to the extent that all of one type of unit may be directed against all of one of the enemy's types of units. Re-allocation is permitted between engagements, but reinforcement is not. All battles are fought to a conclusion; victory is determined by the outcome at the conclusion.



# Ancient Empires

## Model Characteristics

Terrain /Space Abstracted

Time Abstracted as Moves or Engagements

Logistics Hidden

Square Law Attrition

Limited Force Allocation in Battle

Battle to Conclusion

Victory to Complete Conclusion

## ANCIENT EMPIRES

The attrition matrix for a defender force in units killed per attacker per engagement is depicted. In an engagement between cavalry and swordsmen, each cavalry unit kills 1.40 units of swordsmen, while each swordsman unit kills 0.40 units of cavalry.

Examination of this table shows that the cavalry units are the best attriters, while the archers are the worst. In fact, the attrition rates are so unbalanced that an army consisting only of cavalry would be better than any mixed army half again its size.

# Ancient Empires

| Attacker\ Defender | Cavalry | Archers | Swordsmen |
|--------------------|---------|---------|-----------|
| Cavalry            | 0.80    | 1.80    | 1.40      |
| Archers            | 0.07    | 0.40    | 0.30      |
| Swordsmen          | 0.40    | 0.70    | 0.60      |

Defender's losses in kills per attacker per engagement

### ANCIENT EMPIRES

If we take a sample battle with forces as shown here, and use game theory to calculate the optimal strategy for the first engagement, ...

# Ancient Empires

|           | Blue | Red |
|-----------|------|-----|
| Cavalry   | 5    | 10  |
| Archers   | 10   | 12  |
| Swordsmen | 15   | 18  |

Sample Battle

Ancient Empires 5

## ANCIENT EMPIRES

...This is a payoff matrix for the two forces using Loss Exchange Ratio as the payoff function. The maximum values of each row are given in the last column; the minimum values of each column are given in the bottom row.

The strategies for each force are given in the first three columns/rows.

The first row/column is the cavalry strategy, the second is the archer strategy, and the third is the swordsmen strategy. The number refers to which type of enemy force is attacked: 1 = cavalry; 2 = archers; 3 = swordsmen. We see immediately from the Loss Exchange Ratio payoff matrix that the best strategy for Blue is to have his cavalry and archers attack Red's cavalry while his swordsmen attack Red's swordsmen. This minimizes the Loss Exchange Ratio (minimizes Blue losses). Red maximizes the Loss Exchange Ratio by adopting the same strategy.

# Ancient Empires

|   |   |   |   |      |      |      |      |      |      |      |      |      |
|---|---|---|---|------|------|------|------|------|------|------|------|------|
| C | 1 | 2 | 3 | 1    | 2    | 3    | 1    | 2    | 3    | 1    | 2    | 3    |
| A | 1 | 1 | 1 | 2    | 2    | 2    | 3    | 2    | 2    | 3    | 3    | 3    |
| S | 3 | 3 | 3 | 3    | 3    | 3    | 3    | 3    | 3    | 3    | 3    | 3    |
| 1 | 1 | 3 | 3 | 1.08 | 0.90 | 0.97 | 1.00 | 0.84 | 0.90 | 1.02 | 0.86 | 0.92 |
| 2 | 1 | 3 | 3 | 1.18 | 0.98 | 1.05 | 1.10 | 0.92 | 0.98 | 1.12 | 0.94 | 1.01 |
| 3 | 1 | 3 | 3 | 1.14 | 0.95 | 1.02 | 1.06 | 0.89 | 0.95 | 1.08 | 0.91 | 0.97 |
| 1 | 2 | 3 | 3 | 1.15 | 0.96 | 1.02 | 1.06 | 0.90 | 0.96 | 1.09 | 0.91 | 0.98 |
| 2 | 2 | 3 | 3 | 1.25 | 1.04 | 1.11 | 1.16 | 0.97 | 1.04 | 1.18 | 0.99 | 1.06 |
| 3 | 2 | 3 | 3 | 1.21 | 1.01 | 1.08 | 1.12 | 0.94 | 1.01 | 1.15 | 0.96 | 1.03 |
| 1 | 3 | 3 | 3 | 1.13 | 0.94 | 1.01 | 1.04 | 0.88 | 0.94 | 1.07 | 0.90 | 0.96 |
| 2 | 3 | 3 | 3 | 1.23 | 1.02 | 1.10 | 1.14 | 0.96 | 1.02 | 1.16 | 0.98 | 1.04 |
| 3 | 3 | 3 | 3 | 1.19 | 0.99 | 1.06 | 1.10 | 0.93 | 0.99 | 1.13 | 0.95 | 1.01 |
|   |   |   |   | 1.08 | 0.90 | 0.97 | 1.00 | 0.84 | 0.90 | 1.02 | 0.86 | 0.92 |

## Loss Exchange Ratio

Ancient Empires 3

### ANCIENT EMPIRES

If we examine the Conclusion Condition payoff matrix, however, we see a different strategy. Blue may minimize the Conclusion Condition contribution for Red (lengthen the battle--give Red more opportunities for mistakes) by directing his cavalry and archers against Red's archers and his swordsmen against Red's swordsmen. Red's strategy is again identical to Blue's.

This example is essentially the same as the Col. Blotto game, except that optimal strategies are clearer despite the complication of the greater number of strategies. Note that there are no degenerate strategies; all of the strategies are independent. Note also that since the payoff function is symmetric in the two sides, the strategies are identical for both sides. This demonstrates one of the limitations of game theory in tactical use.



# Ancient Empires

|   |   |   |        |        |        |        |        |        |        |        |        |        |
|---|---|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| C | 1 | 2 | 3      | 1      | 2      | 3      | 1      | 2      | 3      | 1      | 2      | 3      |
| A | 1 | 1 | 1      | 2      | 2      | 2      | 3      | 2      | 3      | 3      | 3      | 3      |
| S | 3 | 3 | 3      | 3      | 3      | 3      | 3      | 3      | 3      | 3      | 3      | 3      |
| 1 | 1 | 3 | 122.48 | 222.48 | 182.48 | 170.00 | 270.00 | 230.00 | 155.60 | 255.60 | 215.60 | 270.00 |
| 2 | 1 | 3 | 97.48  | 197.48 | 157.48 | 145.00 | 245.00 | 205.00 | 130.60 | 230.60 | 190.60 | 245.00 |
| 3 | 1 | 3 | 107.48 | 207.48 | 167.48 | 155.00 | 255.00 | 215.00 | 140.60 | 240.60 | 200.60 | 255.00 |
| 1 | 2 | 3 | 89.48  | 189.48 | 149.48 | 137.00 | 237.00 | 197.00 | 122.60 | 222.60 | 182.60 | 237.00 |
| 2 | 2 | 3 | 64.48  | 164.48 | 124.48 | 112.00 | 212.00 | 172.00 | 97.60  | 197.60 | 157.60 | 212.00 |
| 3 | 2 | 3 | 74.48  | 174.48 | 134.48 | 122.00 | 222.00 | 182.00 | 107.60 | 207.60 | 167.60 | 222.00 |
| 1 | 3 | 3 | 99.48  | 199.48 | 159.48 | 147.00 | 247.00 | 207.00 | 132.60 | 232.60 | 192.60 | 247.00 |
| 2 | 3 | 3 | 74.48  | 174.48 | 134.48 | 122.00 | 222.00 | 182.00 | 107.60 | 207.60 | 167.60 | 222.00 |
| 3 | 3 | 3 | 84.48  | 184.48 | 144.48 | 132.00 | 232.00 | 192.00 | 117.60 | 217.60 | 177.60 | 232.00 |
|   |   |   | 64.48  | 164.48 | 124.48 | 112.00 | 212.00 | 172.00 | 97.60  | 197.60 | 157.60 |        |

## Conclusion Condition

## Ancient Empires 4

## OUTLINE

We now reach the last section of this report, that dealing with combat models. In this section we shall be concerned primarily with the basic Lanchester attrition equations for homogeneous forces and to a certain extent with the conjugate theory of the attrition rates. We will not deal with any of the elaborations beyond the basic attrition equations, nor with either heterogeneous forces or such topics as Osipovian attrition.

# OUTLINE

1. Definitions and Background
2. Examples of Games
- 3. Combat Models**

### FACTORS OF WAR

We list some of the factors of war. Of these, the most important is the moral influence: the morale and readiness of the troops themselves. This is the most difficult of the factors to model and simulate--so difficult, in fact, that most simulations do not even treat it because of the lack of confidence in the validity of the models.

The other factors are more straightforward. We shall discuss the effects of weather in some detail; consideration of the others is left as an exercise.

# Factors of War

Moral Influence

Weather

Terrain

Command

Doctrine

### PROCESSES OF COMBAT

There are fundamentally four processes to combat. Almost all of our discussion in the rest of this report will deal with the first, attrition.

This attention is not meant to imply that movement, intelligence and control, and support (including logistics) are not important, merely that any detailed consideration of these factors is beyond the scope of this report.

# Processes of Combat

Attrition

Movement

C3I

Support

**"WEAPONS ARE OMINOUS TOOLS TO BE USED ONLY  
WHEN THERE IS NO ALTERNATIVE"**

This quote from the grandfather of combat theorists tells us two reasons for combat modeling and simulation: first, we may avoid the use of weapons by avoiding war--this goal can be aided by simulation of war so that we may know the mettle of our forces without using them; second, by having a full understanding of our weapons, we may avoid having to ever use them without exhausting all other alternatives.



*"Weapons are ominous tools to be used only  
when there is no alternative."*

Sun Tzu

## LANCHESTER LAW ASSUMPTIONS

### Linear Law Assumptions

This chart outlines the assumptions most commonly associated with the Linear Lanchester Law. There are another set of assumptions which are less commonly used, but these will not be discussed. The assumptions listed are quoted from Kerr.

The first assumption is at once obvious and simple. The premise that the Lanchester Laws model combat is only supportable if combat is indeed occurring between the forces modeled.

The second assumption is open to some discussion. Units in Lanchestrian terms are not the same as units in the normal military usage. Military usage units are a military organization of some given size, complexity, and mission--such as a company or a division. Lanchestrian units are units of combat force or power. (The term unit is used in the Lanchestrian sense much as the term is used in the statement "The dollar is the unit of American currency".) This distinction is a common source of confusion.

The third assumption is also open to discussion. It is easy to argue that attrition rates should be functions and not just constants. We will consider this in greater detail later. The calculation of attrition rates has also been a problem area; it has only been in recent years that a conjugate theory of attrition rates has begun to evolve.

The fourth assumption states that while the positions of the targets are known to the firing units, the effect of that fire is not well known.

Finally, the fifth assumption states that fire is apportioned evenly over the area occupied by the enemy.

The last two assumptions give rise to the common interpretation that the Linear Lanchester Law represents combat with area weapons such as artillery.

# Lanchester Law Assumptions

*Most Common Use*

## Linear Law Assumptions

1. Combat occurs between two forces.
2. All units are within weapon range of each other.
3. Attrition rates are constant and known.
4. Position intelligence is good, but casualty intelligence is poor.
5. Fire is uniformly distributed over enemy occupied area.

### **LANCHESTER LAW ASSUMPTIONS** **Quadratic Law Assumptions**

This chart presents the most common assumptions associated with the Square (or Quadratic) Lanchester Law. The first three assumptions are the same as those given in the previous slide, so we shall skip immediately to the last two assumptions.

In this case, intelligence of the enemy units' positions and the effect of fire on them is known, and fire is apportioned among the surviving enemy units. These two assumptions lead to the interpretation that the Quadratic Lanchester Law represents combat with point fire weapons such as tanks.

# Lanchester Law Assumptions

*Most Common Use*

## Quadratic Law Assumptions

1. Combat occurs between two forces.
2. All units are within weapon range of each other.
3. Attrition rates are constant and known.
4. Position and casualty intelligence are good.
5. Fire is uniformly distributed over surviving enemy units.

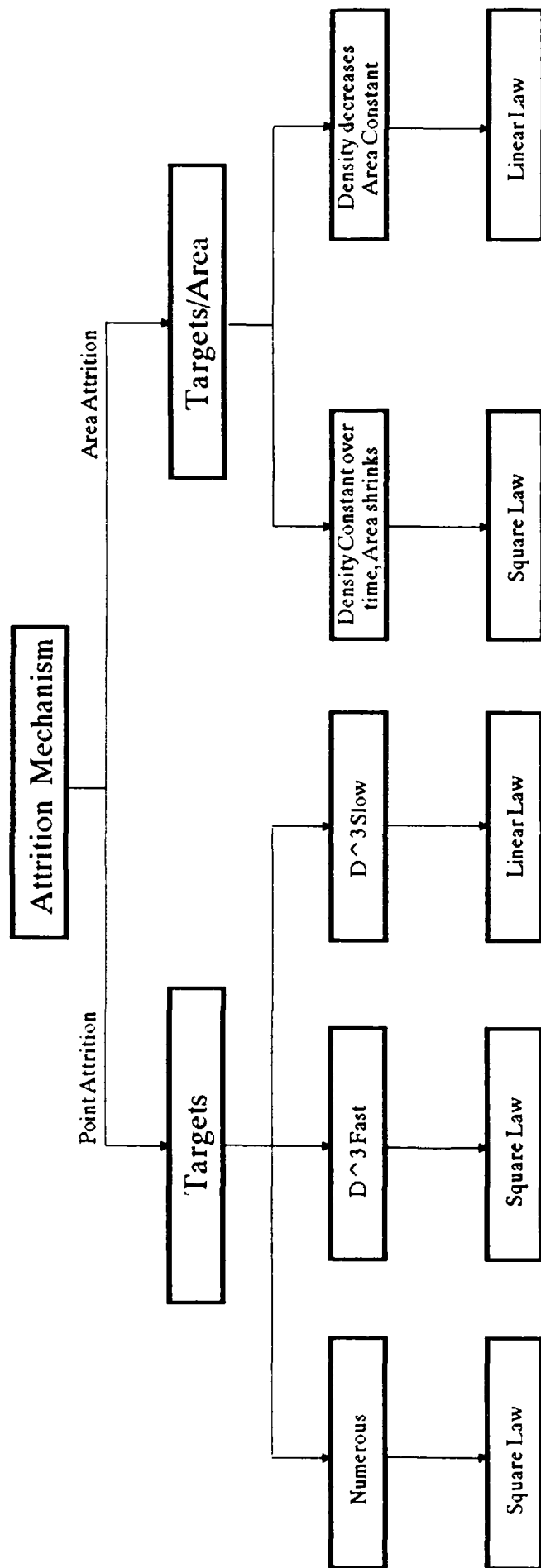
### ATTRITION LAW DECISION TREE

A decision tree may be defined for the two Lanchester Laws. If the weapon being used is a point fire weapon, or individual targets are being attacked, then the left hand branch (from the top) is used. If the weapon being used is an area fire weapon, or targets spread over an area are being attacked, then the right hand branch is used.

Within the right hand branch, if the target units are numerous, or the rate of attrition is high (attrition occurs quickly), then the Square Law applies. (We note that numerous targets and rapid attrition are equivalent concepts in Lanchestrian terms.) If the rate of attrition is low (or targets are few), then the Linear Law applies.

Within the left hand branch, if the number of targets per area remains constant over time while the occupied area (usually) shrinks (occasionally expands), then the Square Law applies. If the number of targets changes (usually decreases), but the occupied area remains constant, then the Linear Law applies.

# Attrition Law Decision Tree



### LANCHESTER LAW DIFFERENTIAL EQUATIONS

Lanchester Laws may be expressed in differential equations in two forms: the Linear and Quadratic Lanchester Laws. The quantities A and B are the force strengths expressed in units. The greek letters alpha and beta are the attrition rates expressed in either units killed per time per enemy unit per friendly unit (Linear Law), or units killed per time per enemy unit.

Note that these are coupled first- order differential equations.



# Lanchester Law Differential Equations

## Linear Law

$$\frac{dA}{dt} = -\alpha A, \quad \frac{dB}{dt} = -\beta B$$

## Quadratic Law

$$\frac{dA}{dt} = -\alpha A^2, \quad \frac{dB}{dt} = -\beta B^2$$

$\alpha, \beta$  are attrition rates.

$A, B$  are Force Strengths.

## STATE SPACE SOLUTIONS

### Linear Equations

The state solutions of the differential equations are obtained by removing time as an independent variable. These solutions are usually very easy to obtain, and are at least as old as the differential equations themselves. Lanchester quotes both state solutions in his book.

This chart presents the Linear Law state solution. This solution has the form of a linear equation in the force strengths (the zero subscript terms are the initial force strengths at the start of the combat); hence the term Linear Law.

# State Space Solutions

## Linear Equations

$$\beta (A - A_0) = \alpha (B - B_0)$$

# State Space Solutions

## Quadratic Equations

$$\alpha - \frac{(A^2 - A_0^2)}{2} = \frac{\beta}{2} (B^2 - B_0^2)$$

## STATE SPACE SOLUTIONS

### Quadratic Equations

The Quadratic (or Square) Law state solution is given here. This solution has the form of a quadratic equation (the force strengths appear as squared terms); hence the term Quadratic Law.

## STATE SPACE SOLUTIONS

### Linear Equations

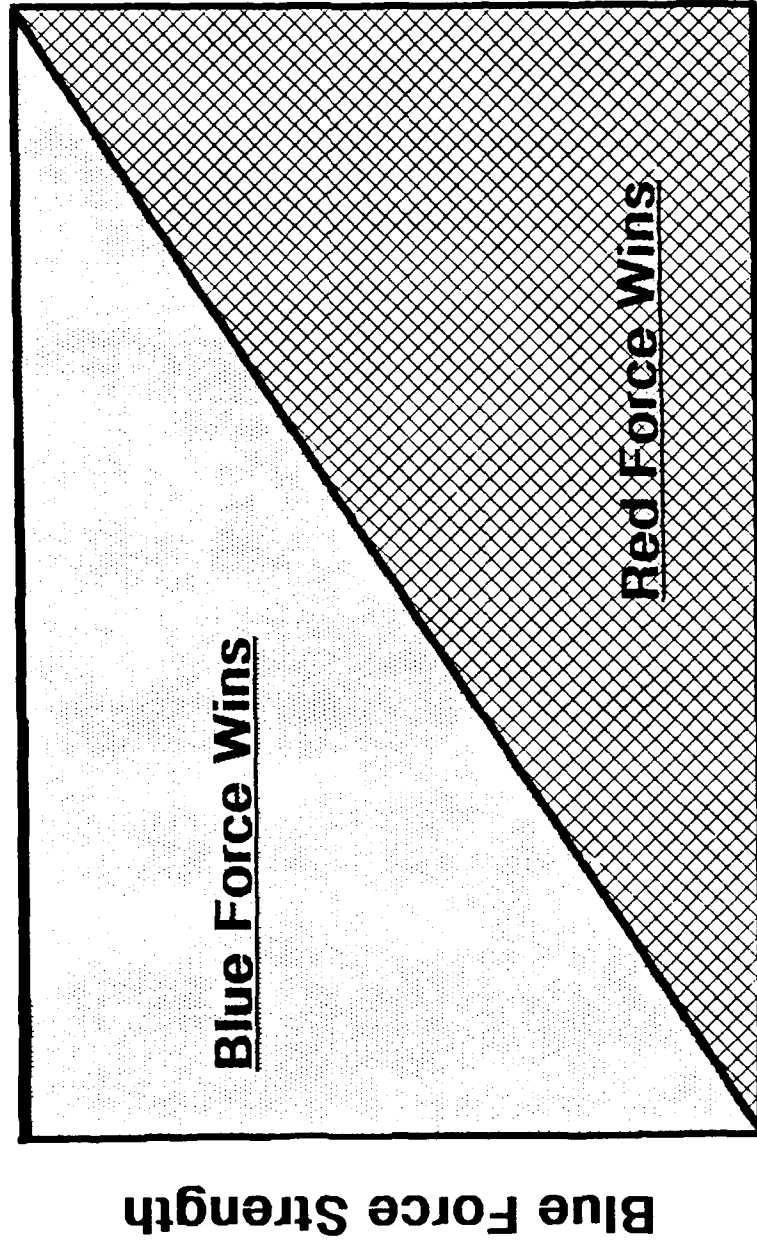
A special case of the state solutions is called the draw case. The state solutions represent combat to a conclusion. Lanchester introduced the term "conclusion" to indicate combat to the point where units of only one force survive. The draw case occurs when both forces are completely destroyed (attrited) if combat continues to a conclusion. If we plot the Blue force strength along the vertical axis and the Red force strength along the horizontal axis, then the draw case is the straight line drawn diagonally across the graph.

This line divides the graph (called a state space plot) into two halves. State solutions which plot above the draw line will be Blue victories if the combat is carried to a conclusion; solutions which plot below the draw line will be Red victories if combat is carried to a conclusion.

# State Space Solutions

*State Space Plot*

## Linear Equations



Red Force Strength

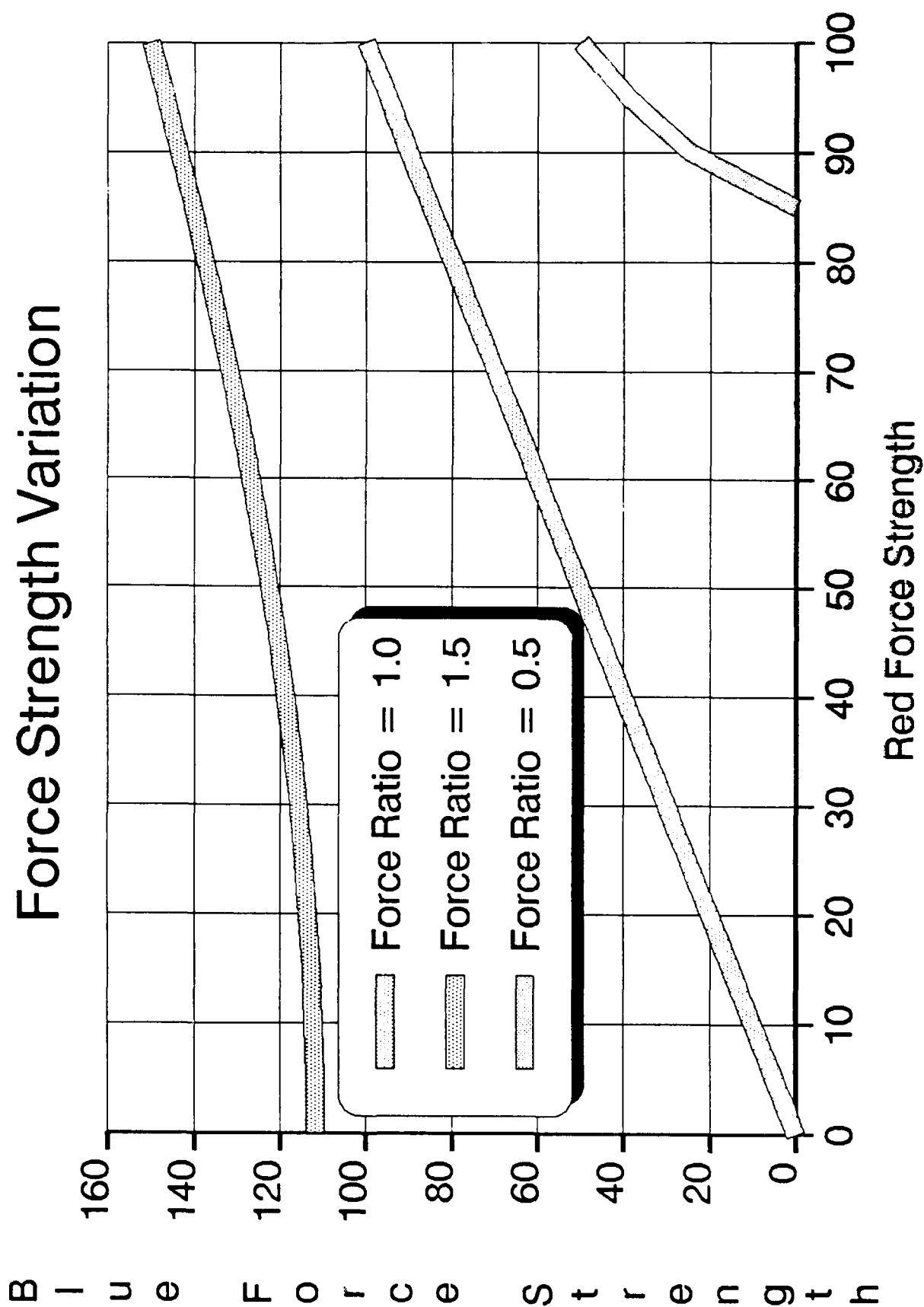
## QUADRATIC LANCHESTER EQUATION

### Force Strength Variation

This graph depicts the effect on the Quadratic Law state solution with change in initial force strength (the attrition rates remain fixed). The center line is the draw line. The lower line represents the state solution when the Red force has its strength doubled (compared to the draw case). The upper line represents the state solution when the Blue force has its strength doubled.



# Quadratic Lanchester Equation



## QUADRATIC LANCHESTER EQUATION

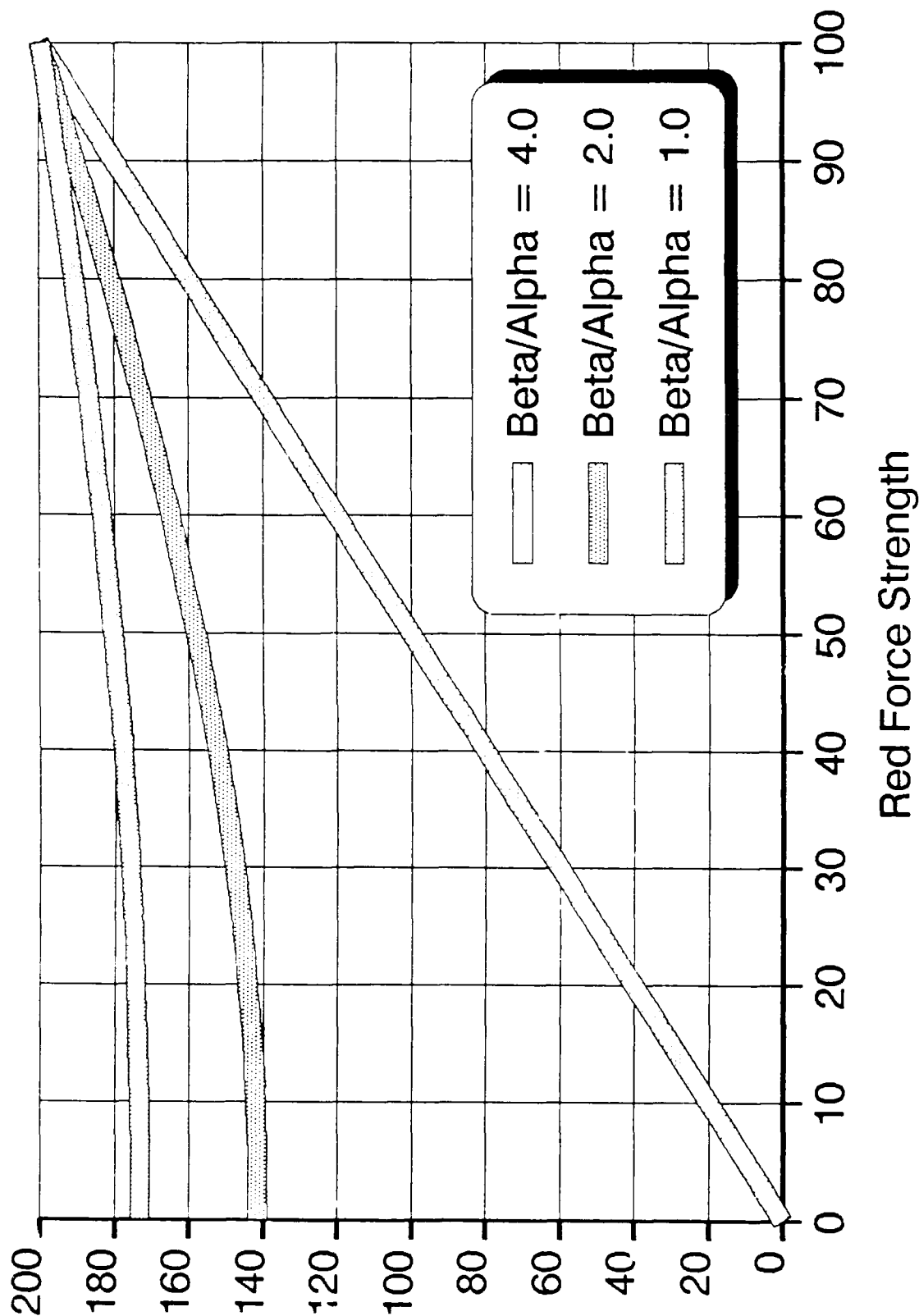
### Attrition Rate Variation

This graph depicts the effect on the Quadratic Law state solution with change in the attrition rates. The lower curve is the draw case. The two upper curves represent doubling and quadrupling of the Blue attrition rate relative to the draw case.

# Lanchester Quadratic Equation

Blue Force Strength

Attrition Rate Variation



## TIME SOLUTIONS

### Linear Equations

Explicit time solutions for the Linear Law are presented. This is a fairly recently discovered solution, first appearing in the literature in the 1950s.

The quantity delta is the residual combat power remaining to the victorious force if combat continues to a conclusion. Combat power in this case is the number of kills per time per enemy unit. The sign of this parameter indicates the victor (if combat is carried to a conclusion). If delta is positive, the Red force (A) will be the victor; if negative, Blue (B) will be the victor. This parameter is zero for the draw case.

# Time Solutions

## Linear Equations

$$A(t) = A_0 \frac{\Delta_1}{\beta A_0 - \alpha B_0 e^{-\Delta_1 \Delta t}}$$

$$B(t) = B_0 \frac{-\Delta_1}{\alpha B_0 - \beta A_0 e^{\Delta_1 \Delta t}}$$

$$\beta A_0 - \alpha B_0 \equiv \Delta_1$$

CK

## TIME SOLUTIONS

### Quadratic Equations

There are also explicit time solutions for the Quadratic Law.

This solution has been known since the 1930s in the West, although Osipov presents this solution in his papers (contemporary with Lanchester's book).

The parameter  $\gamma$  is introduced to simplify the notation.

The parameter  $\delta$  is again residual combat power, but in this case is friendly units - kills per time. The sign convention described before also holds.

# Time Solutions

## Quadratic Equations

$$A(t) = A_0 \cosh(\gamma \Delta t) - \left[ \frac{\alpha}{\beta} B_0 \sinh(\gamma \Delta t) \right],$$

$$B(t) = B_0 \cosh(\gamma \Delta t) - \left[ \frac{\beta}{\alpha} A_0 \sinh(\gamma \Delta t) \right],$$

$$\Delta_2 = \alpha B_0^2 - \beta A_0^2$$

$$\alpha \beta = \gamma^2$$

## TIME SOLUTIONS

### Mixed Equations

An interesting excursion of the basic Lanchester Laws is the Mixed Law. In this case, one force loses units as described by the Linear differential equation while the other force loses units as described by the Quadratic differential equation. (The Red force losses obey the Quadratic differential equation, while the Blue force losses obey the Linear differential equation). No published source for these equations is known; their straightforward derivation is due to the author.

While the Quadratic Law is commonly associated with direct fire weapons, and the Linear Law is associated with indirect fire weapons, the Mixed Law is associated with combat between two forces which use both of these types of weapons. For example, the Mixed Law may be used to describe combat between aircraft with bombs (area weapons) and air defense (direct fire) weapons, or between antisubmarine ships (depth charges) and submarines (torpedoes). They also describe direct fire combat when one force is easy to find (rapid attrition) and the other force is hard to find (slow attrition), as is the case when a battle comes close to a conclusion but one force is still large. Another situation described is area fire combat between a force which keeps a constant units-to-area ratio (area shrinks) and a force which keeps a constant area.



# Time Solutions

## Mixed Equations

$$A(t) = \frac{\left[ \frac{\Delta_m}{\beta} \int \beta (1 + e^{-\eta \Delta t}) - \sqrt{\Delta_m} (1 - e^{-\eta \Delta t}) \right]}{A_0 \int \beta (1 - e^{-\eta \Delta t}) + \sqrt{\Delta_m} (1 + e^{-\eta \Delta t})}$$

$$B(t) = B_0 \frac{\Delta_m e^{-\eta \Delta t}}{\left[ \sqrt{\alpha B_0 + \Delta_m/2} (1 - e^{-\eta \Delta t}) + \sqrt{\Delta_m/2} (1 + e^{-\eta \Delta t}) \right]}$$

$$\Delta_m = \beta A_0^2 - 2\alpha B_0$$

CM

### VICTORY PREDICTION

Several comments may be made on victory prediction using the Lanchester Laws, which are frequently used to predict which force will be victorious.

Historical data have been analyzed and all of these analyses indicate that initial force strength has little to do with victory.

Very few battles have ever proceeded to conclusion. On the other hand, there seems to be a strong pattern for the few battles which have proceeded to conclusion.

When one side has a great superiority of units over the other side, the large side has difficulty using all of its units in combat. Thus, the first assumption of the Lanchester Laws may be violated.

# Victory Prediction

- ▶ Lanchester Equation Solutions are often used to predict the outcome of combat (i.e. victory.)
- ▶ Annihilation solutions (combat to a conclusion) are elegant and tantalizing.
- ▶ Analyses of historical data show no strong correlation of force strength and victory.
- ▶ Analyses of historical data also show that (very) few battles proceed to a conclusion.
- ▶ From basic principles, Lanchester Laws are invalid for exceedingly one-sided force ratios, and do not include moral perceptions.

## SHORT BATTLES

### Initial Force Strength vs. Date

This scatter plot presents some data for selected short battles, most less than one day in length. Short battles were selected to avoid the question, to the maximum extent possible, of how long the forces were engaged in combat. Additionally, battles were selected for which five pieces of information were available: initial and final force strengths of both sides, and the duration of the battle. These restrictions reduced a data base of approximately 40,000+ battles to 72.

85

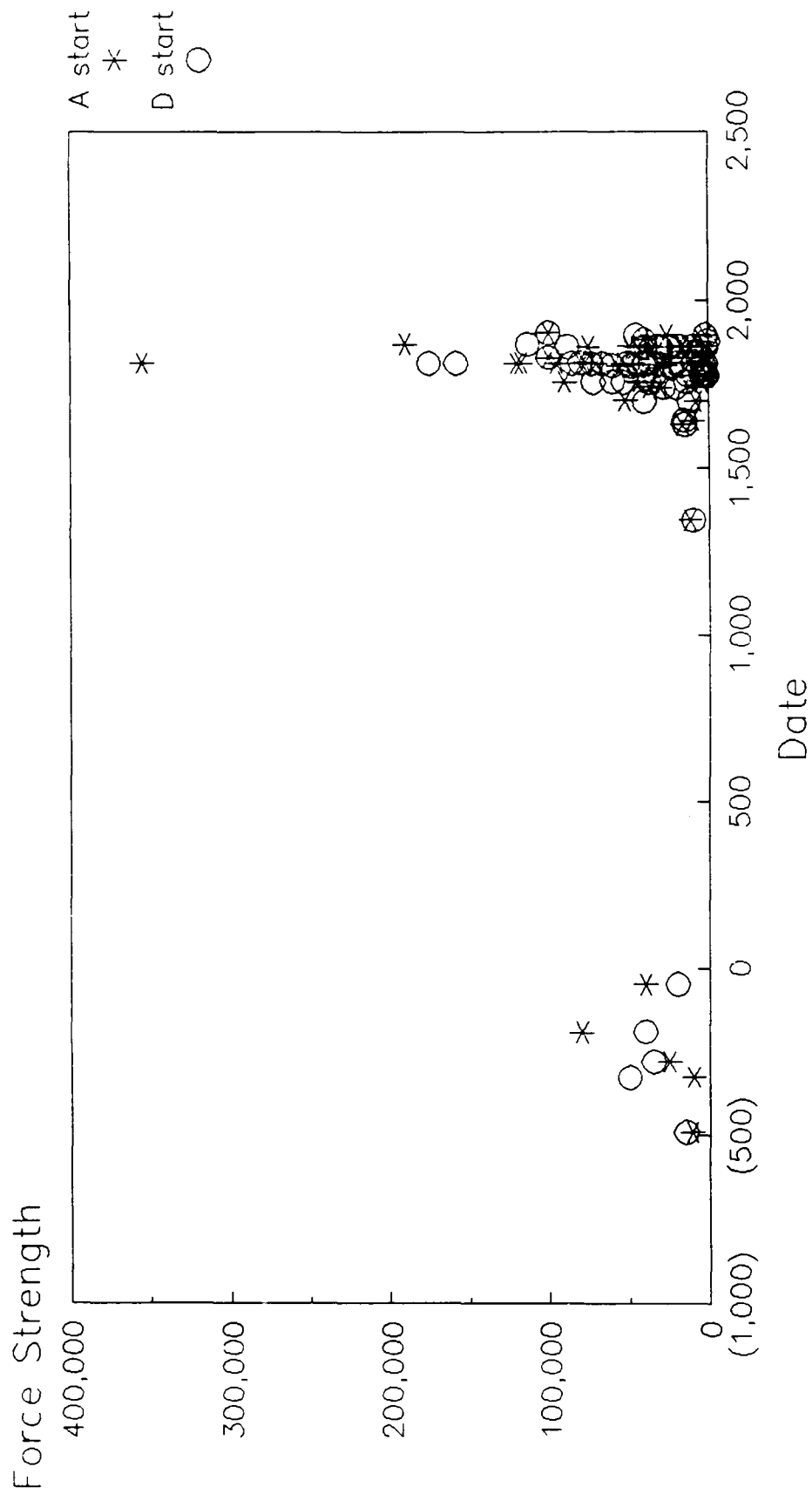
This chart shows that there is no strong correlation between when the battle occurred (date) and the size of the forces involved. It does indicate that armies have become larger since ancient times.

Note that there is very little data from the period of the Middle Ages.

Also, since the 20th Century began, there have been very few short battles. What are called battles today would have been termed campaigns a century ago.

# Short Battles

## Initial Force Strengths vs. Date



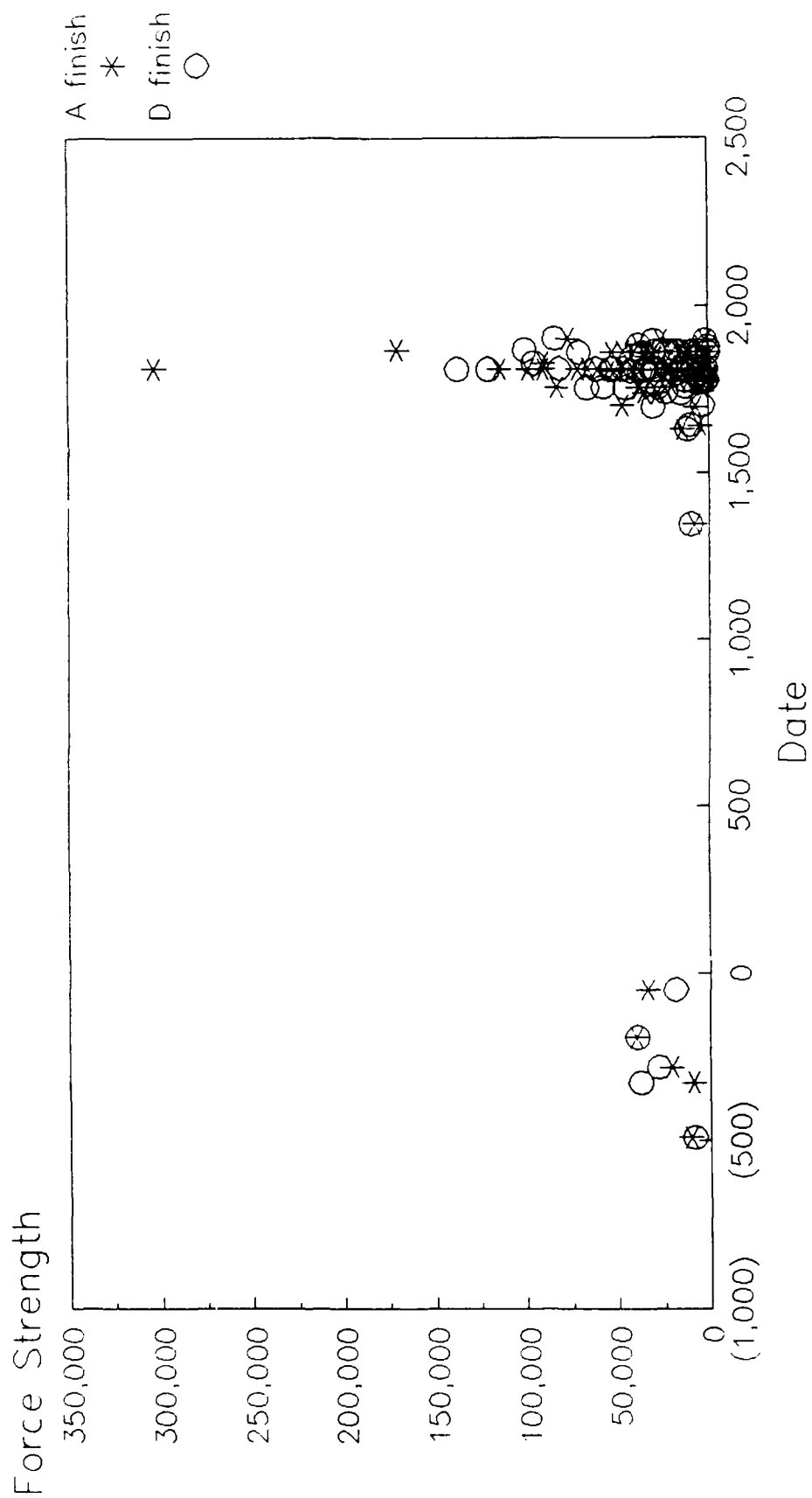
## SHORT BATTLES

### Final Force Strength vs. Date

This scatter plot presents the same type of data as the previous chart except for final force strengths. Note the same lack of correlation.

# Short Battles

## Final Force Strength vs. Date



### SHORT BATTLES

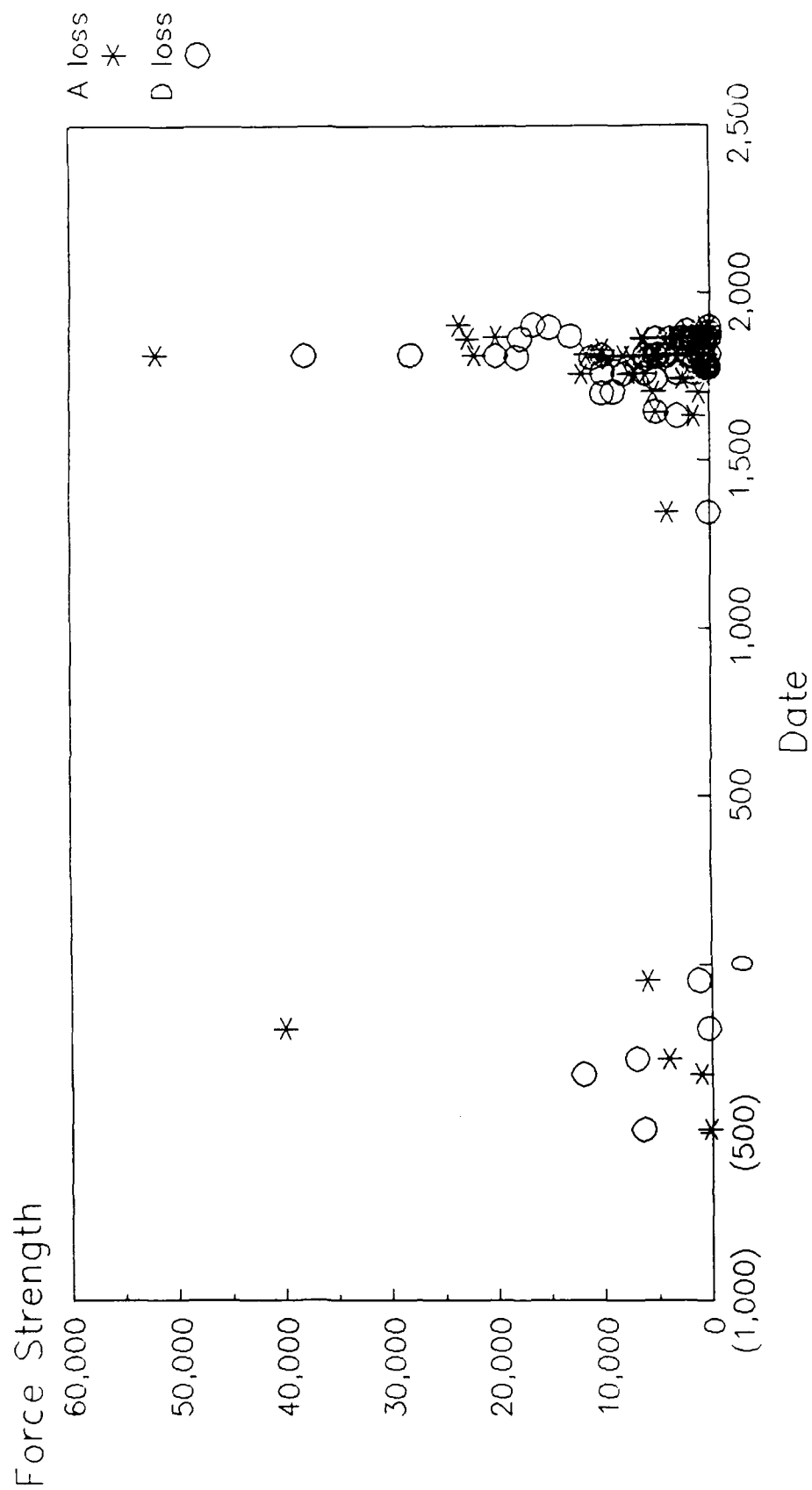
#### Force Strength Losses vs. Date

This plot presents the differences between the two previous plots, losses plotted against date. Again there is no strong correlation.



# Short Battles

## Force Strength Losses vs. Date



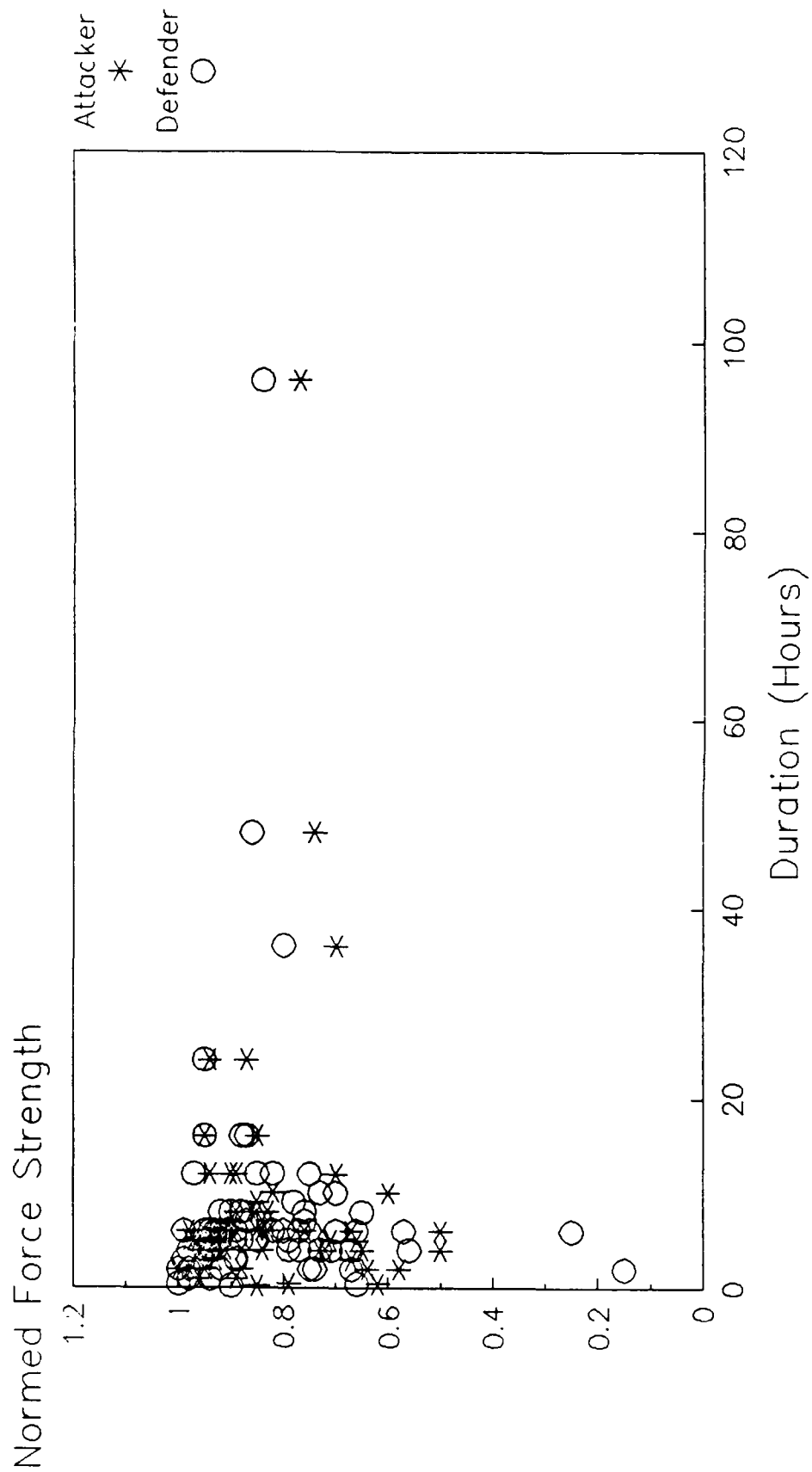
## SHORT BATTLES

### Normed Force Strengths vs. Duration

This scatter plot presents the normed (final divided by initial) force strengths plotted versus duration of the combat. (Duration is estimated from brief historical accounts.) The only correlation to be drawn here is that the battles are short, which was a preselection factor. The mean value of normed force strength appears to be about 0.8. We will explore this in a later chart.

# Short Battles

## Normed Force Strengths vs. Duration



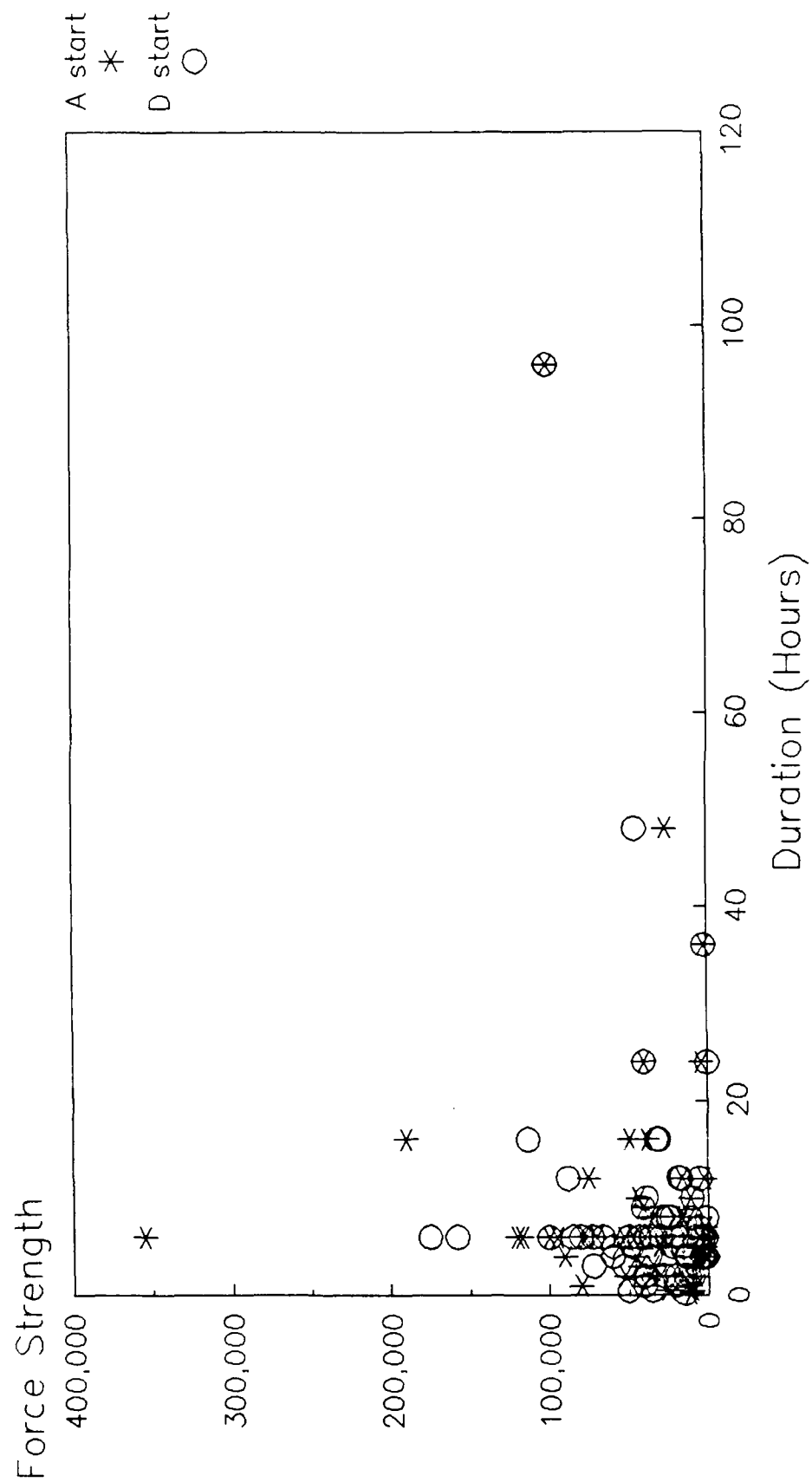
## SHORT BATTLES

### Initial Force Strengths vs. Duration

This scatter plot presents initial force plotted versus duration. Some argument can be made that the data approximate a gamma distribution.

# Short Battles

## Initial Force Strengths vs. Duration



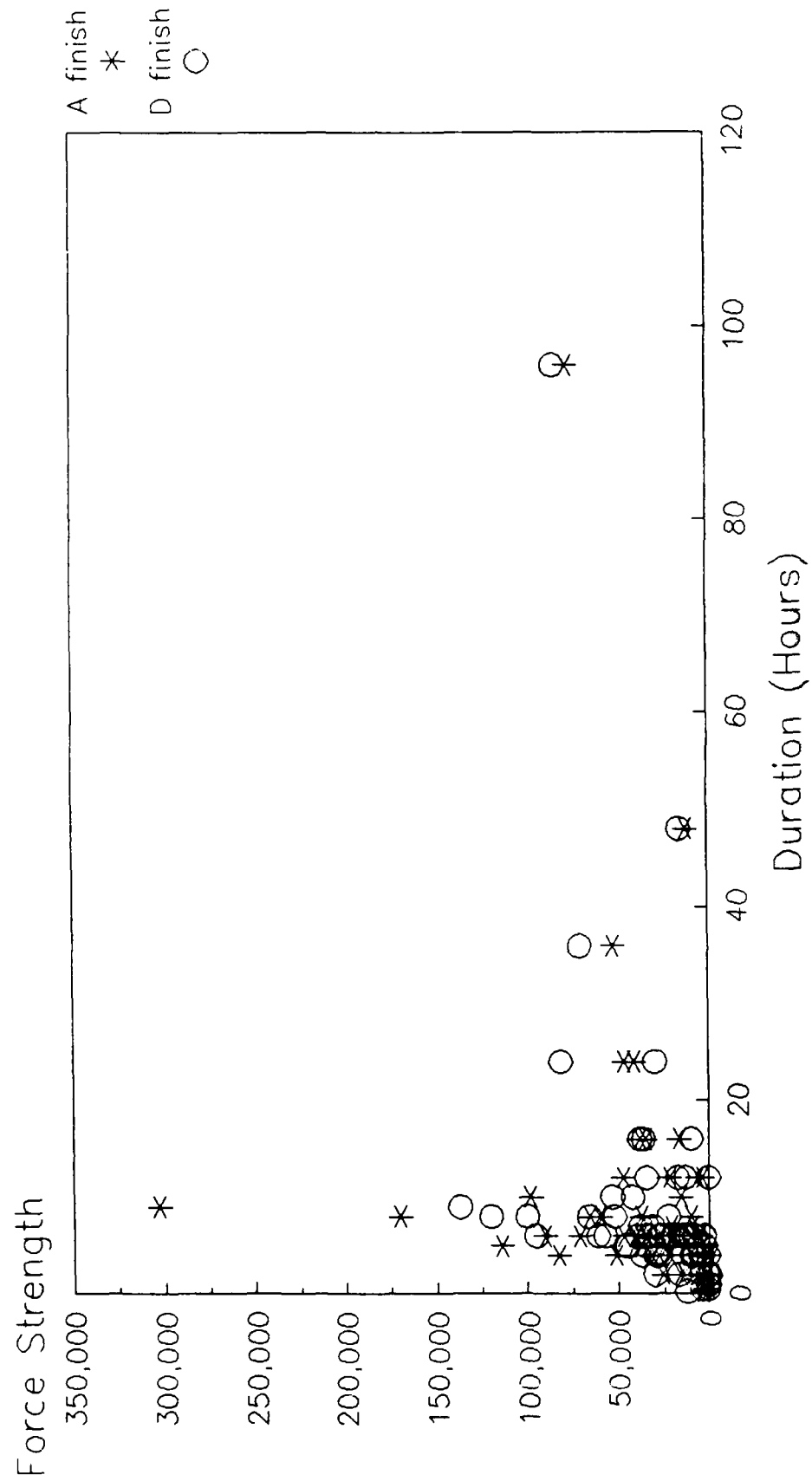
## SHORT BATTLES

### Final Force Strengths vs. Duration

In this case, final force strengths are plotted versus duration.

# Short Battles

## Final Force Strengths vs. Duration



## SHORT BATTLES

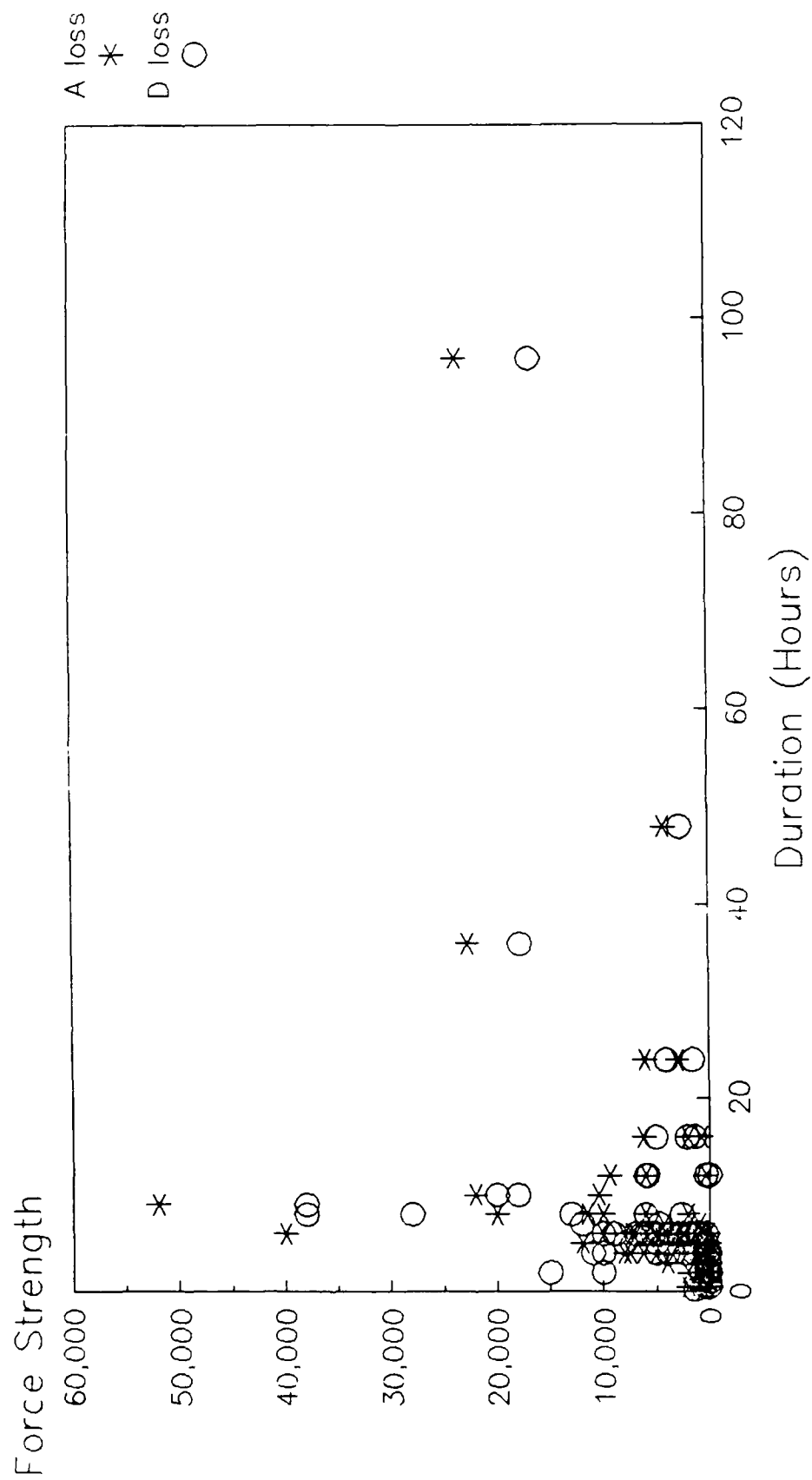
### Loss in Force Strength vs. Duration

This chart plots force strength losses versus duration.



# Short Battles

## Loss in Force Strength vs. Duration



## SHORT BATTLES

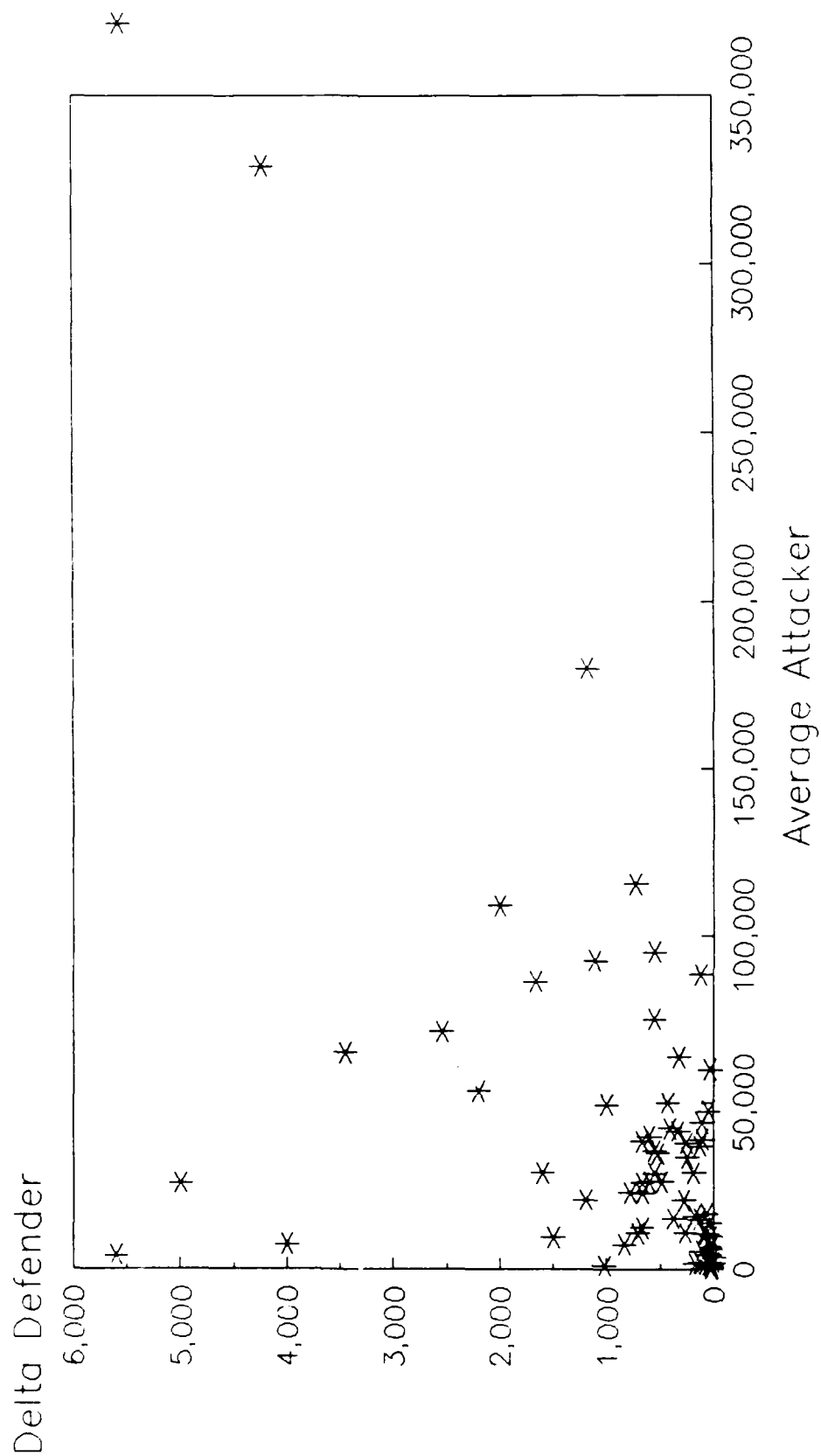
### Delta Defender vs. Average Attacker

This chart plots the Defender's losses versus the average Attacker force strength (initial plus final force strength divided by two). This plot should represent some association with the Quadratic Lanchester differential equation (first-order numeric integration approximation).

Note that there is still no strong correlation.

# Short Battles

## Delta Defender vs. Average Attacker



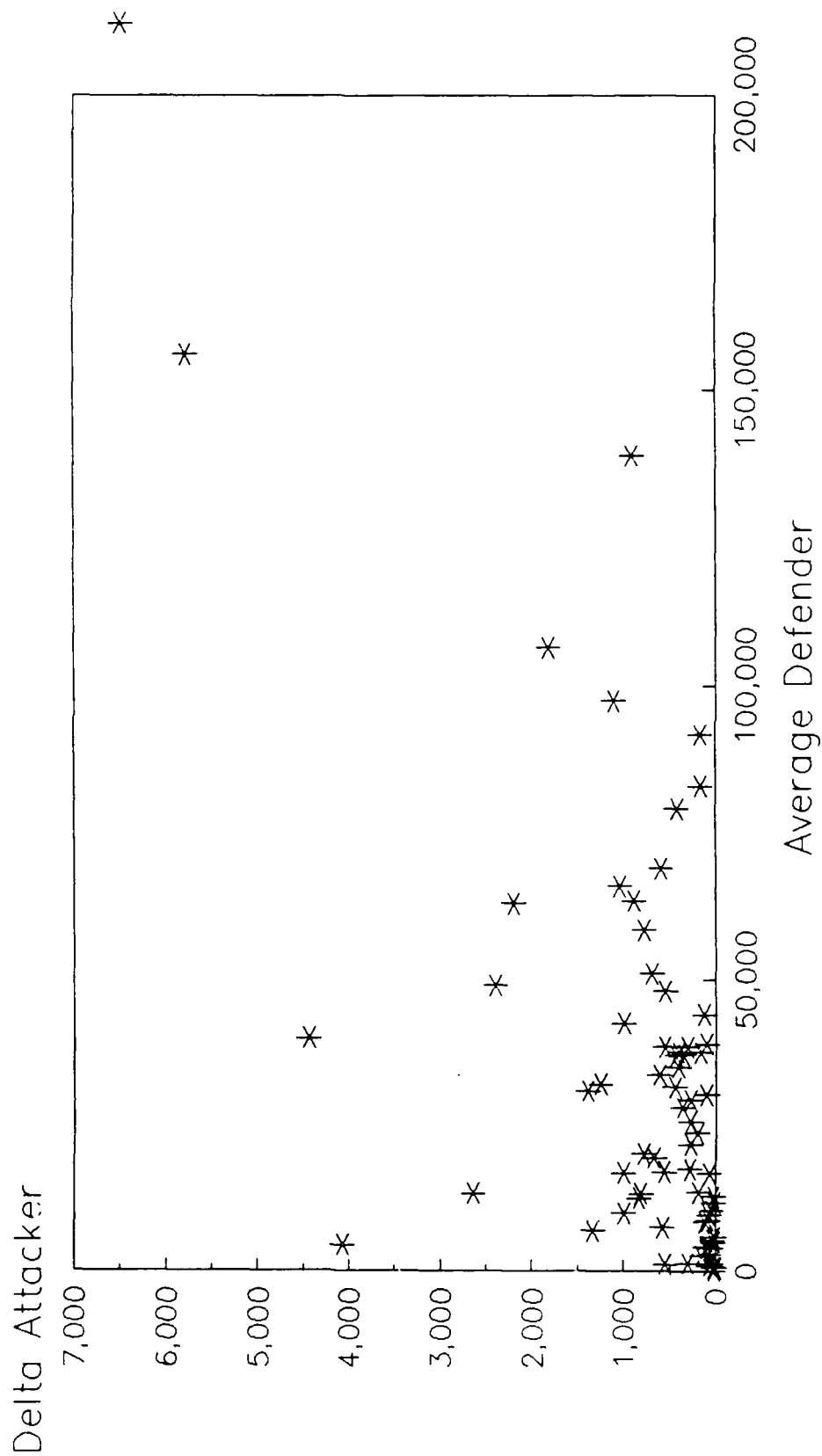
## SHORT BATTLES

### Delta Attacker vs. Average Defender

This plot is the reverse of the previous one. The Attacker's losses are plotted against the average Defender force strength. Again there is no strong correlation.

# Short Battles

## Delta Attacker vs. Average Defender



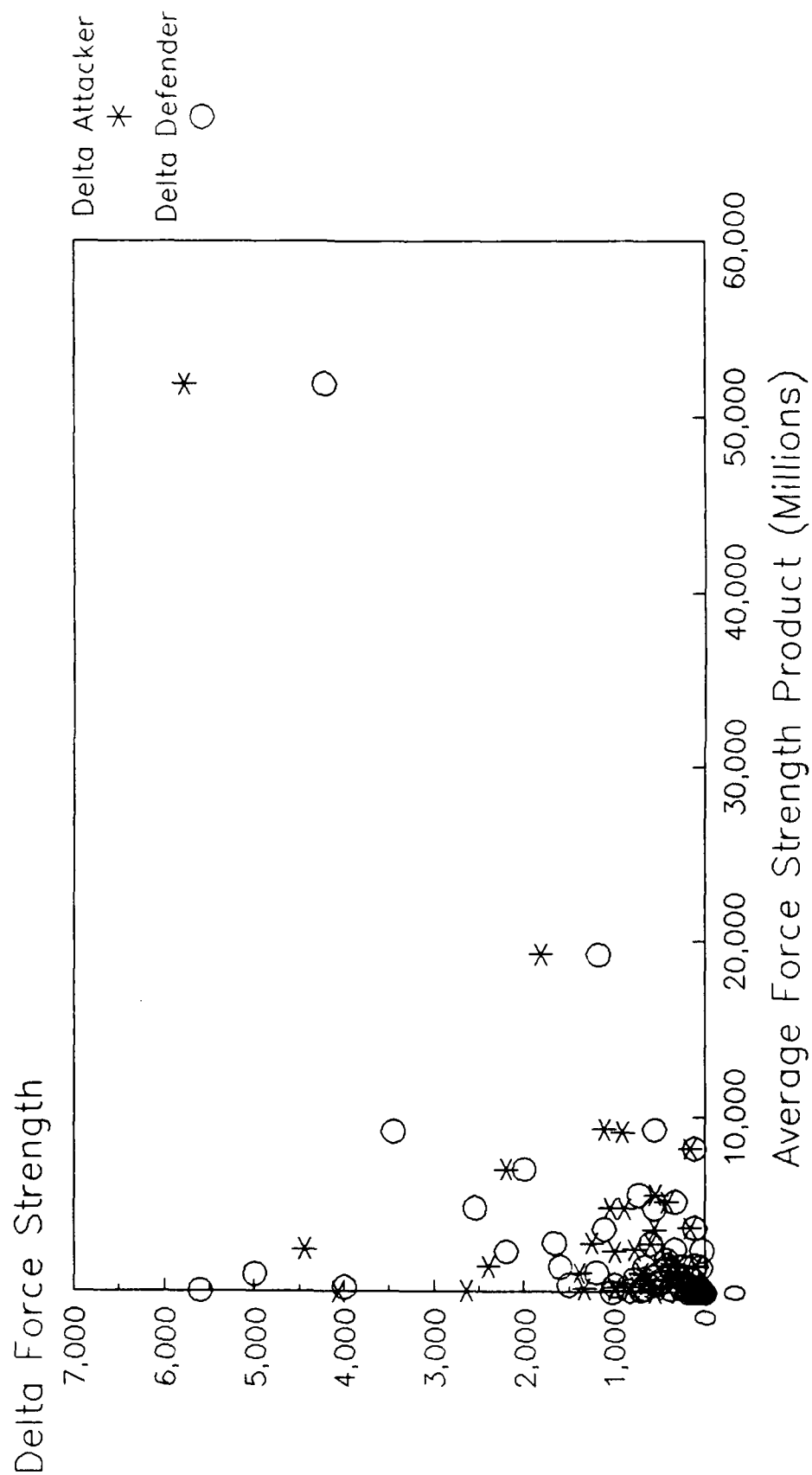
### SHORT BATTLES

#### Delta Force Strength vs. Average Force Strength Product

This plot shows the Attacker's and Defender's losses versus the average of the product of the Attacker's and Defender's force strengths.

This is a first-order numeric integration of the Linear Lanchester differential equation, and should be a test of the correlation of the data with the Linear Law.

## Delta Force Strength vs. Average Force Strength Product



## SHORT BATTLES

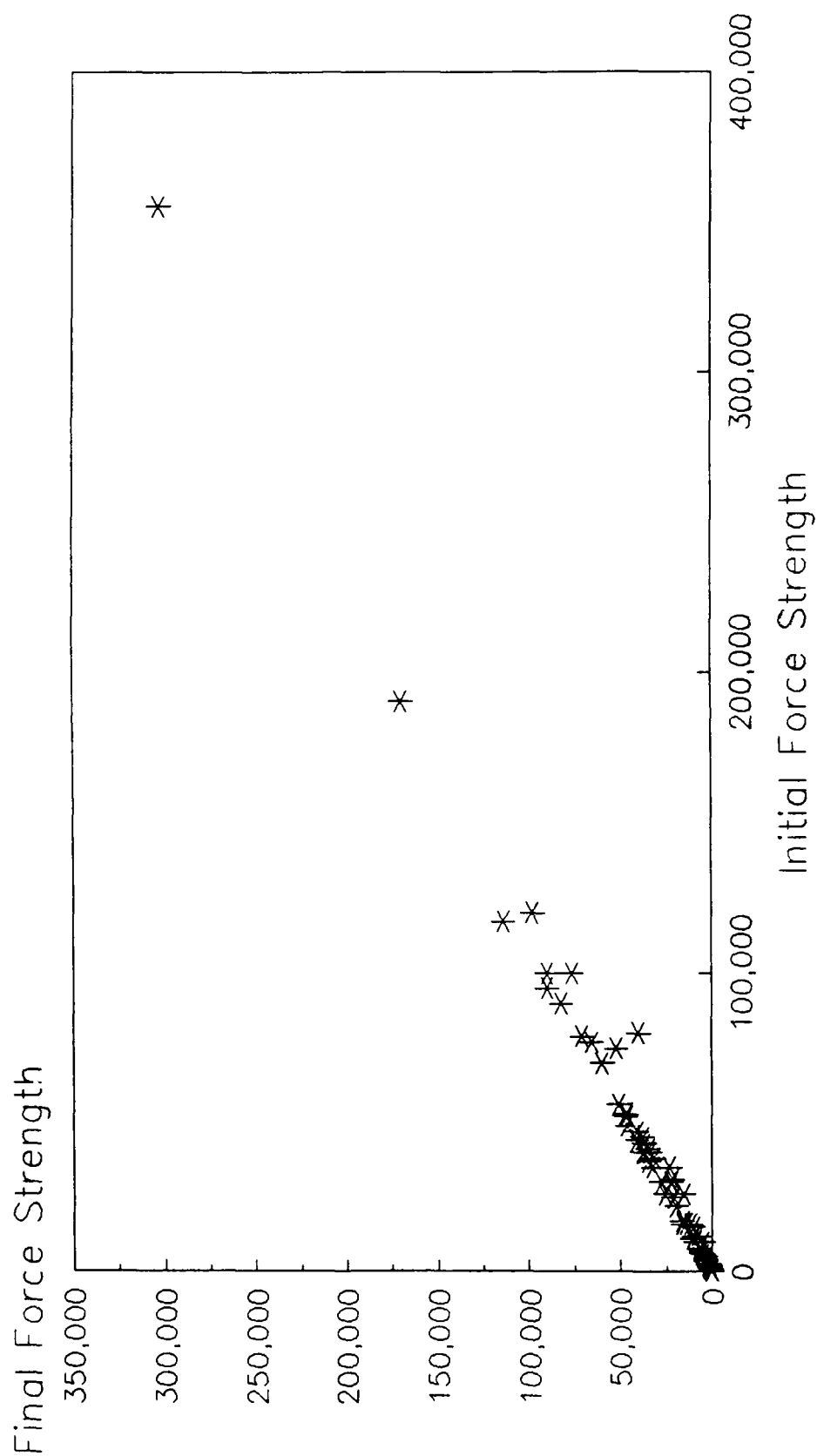
### Final vs. Initial Force Strengths Attacker

This plot shows Attacker's final force strength versus Attacker's initial force strength. Note the strong correlation! This type of behavior is consistent with the Quadratic Lanchester differential equation for the draw case.



# Short Battles

## Final vs. Initial Force Strengths Attacker



## SHORT BATTLES

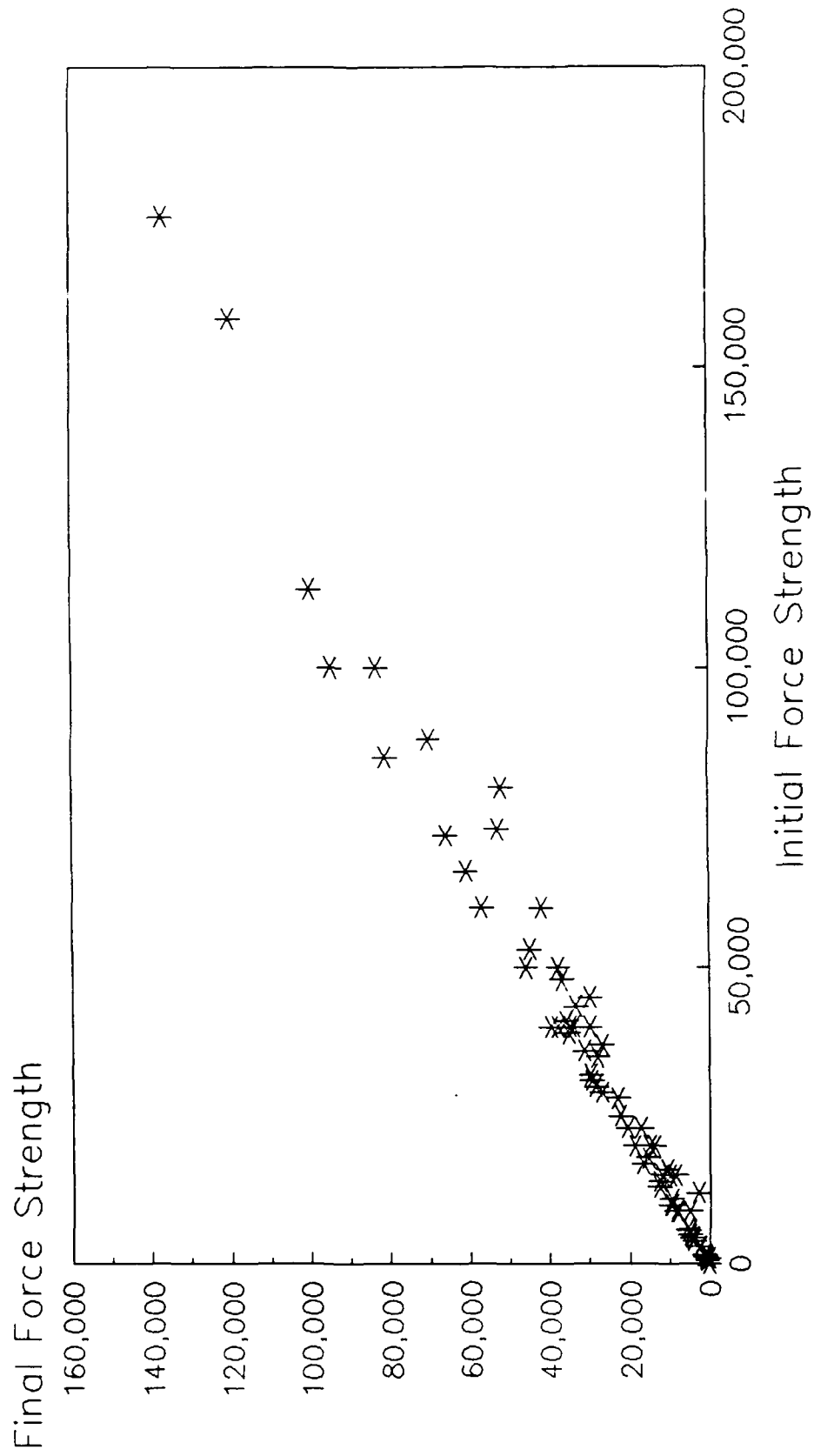
### Final vs. Initial Force Strengths Defender

This plot is the same form as the previous one except for the Defender data.

# Short Battles

## Final vs. Initial Force Strengths

### Defender



## SHORT BATTLES

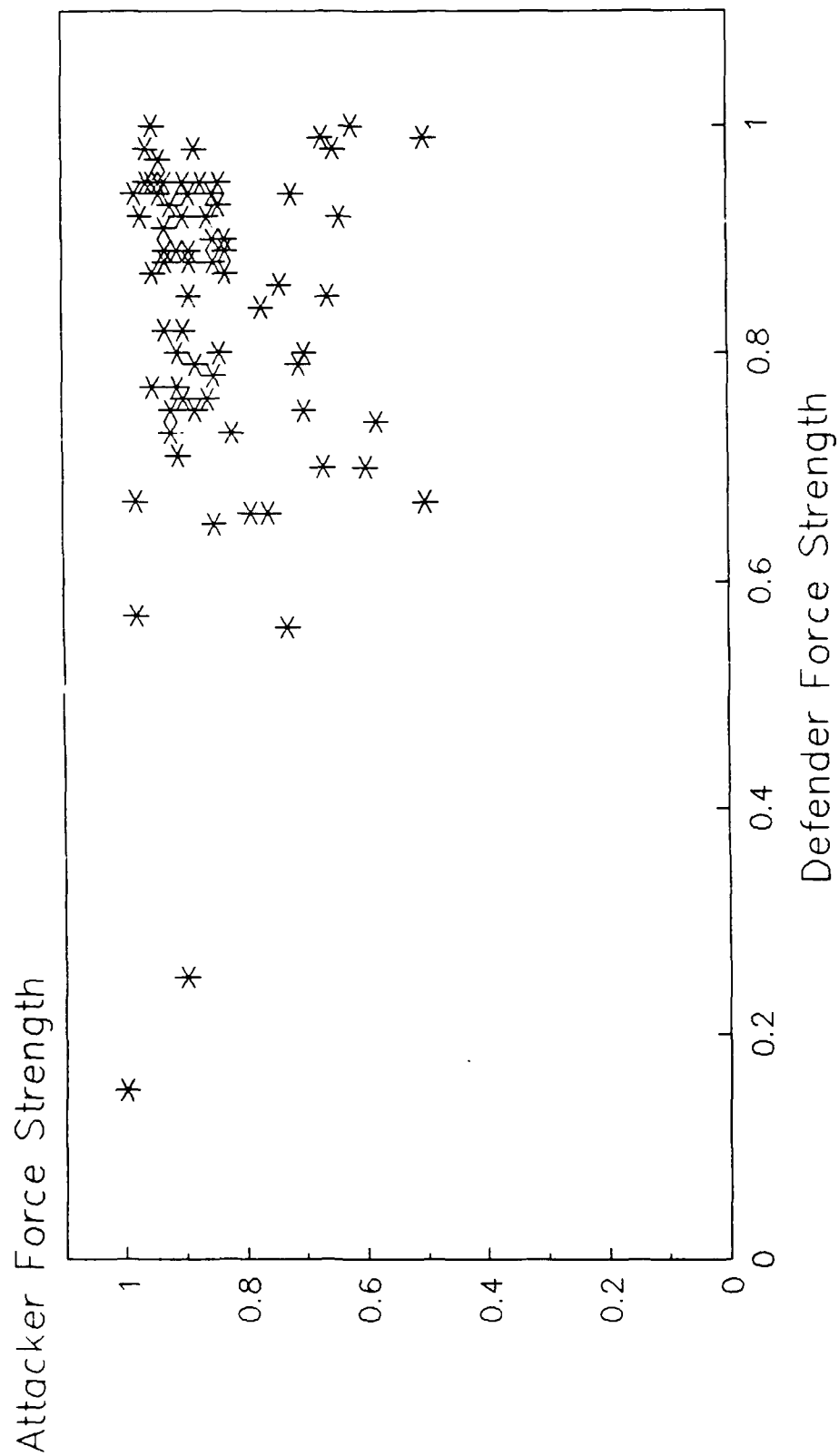
### Attacker vs. Defender Force Ratios

This plot shows the normed Attacker force strength versus the normed Defender force strength. While there is again no strong correlation, note that very little data is shown for normed force strengths less than about 0.6. This is very far from conclusion conditions.

These charts do not provide any firm conclusions, but they do point out the difficulties of using Lanchester equations to predict victory (since few battles are fought to conclusion, indeed most proceed to only slight losses), and that the correlation of history with the Lanchester equations is open to discussion.

# Short Battles

## Attacker vs. Defender Force Ratios



#### GENERAL OBSERVATION ABOUT THE LANCHESTER LAWS

The Lanchester Laws do not include any of the combat processes except attrition in an exhaustive manner. Inclusion of these other processes must await the development of a more comprehensive combat model, ...

Despite popular attempts to the contrary, the Lanchester Laws are not models of the general combat process; they are models of some of the attrition part of the combat process. As such, they are a part of an as-yet incompletely developed conjugate to a transport theory of combat.

## TRANSPORT THEORY OF COMBAT

...Such as the strawman model shown in this chart.



# Transport Theory of Combat

## Transport Equation

$$\frac{\partial B}{\partial t} + \mathbf{v} \cdot \nabla(\mathbf{r}) B + \mathbf{a} \cdot \nabla(\mathbf{v}) B = \frac{dB_{\text{losses}}}{dt}$$

Movement  $\mathbf{v} \cdot \nabla(\mathbf{r}) B$

Other Influences  $\mathbf{a} \cdot \nabla(\mathbf{v}) B$

Attrition  $\frac{dB_{\text{losses}}}{dt}$

## TWO APPROACHES TO ATTRITION PROCESSES

There are two fundamental approaches to combat modeling to understand the attrition process. The first is to use the basic analytical solutions and the second is to use conjugate models which must usually be used as nonanalytic solutions. An attempt to join the two is embodied in the attrition rate theory which is conjugate to Lanchester attrition theory.

## Two Approaches to Attrition Processes

1. Use models which yield closed form simulations. (i.e. either classical Operations Research problems or analytical solutions to Lanchester Differential Equations.)
2. Use conjugate models which usually preclude closed form simulations (i.e. models of physical and/or psychological reality.)

## ATTRITION PROCESS

### Conventional Form

This chart lists the four conventional activities associated with the attrition process. These four are specific to the direct fire case; the fourth activity, target rejection--the decision to find another target--is not usually included in the Square Law.

# Attrition Process

## Conventional Form

1. Target Acquisition
2. Target Selection
3. Target Engagement
4. Target Rejection

## ATTRITION PROCESS

### Smart Munition Form

This chart presents the activities associated with the attrition process in their smart munitions form. While the equivalence is (hopefully) obvious, the question of target rejection becomes moot for smart munitions by their very nature.

# Attrition Process

## Smart Munition Form

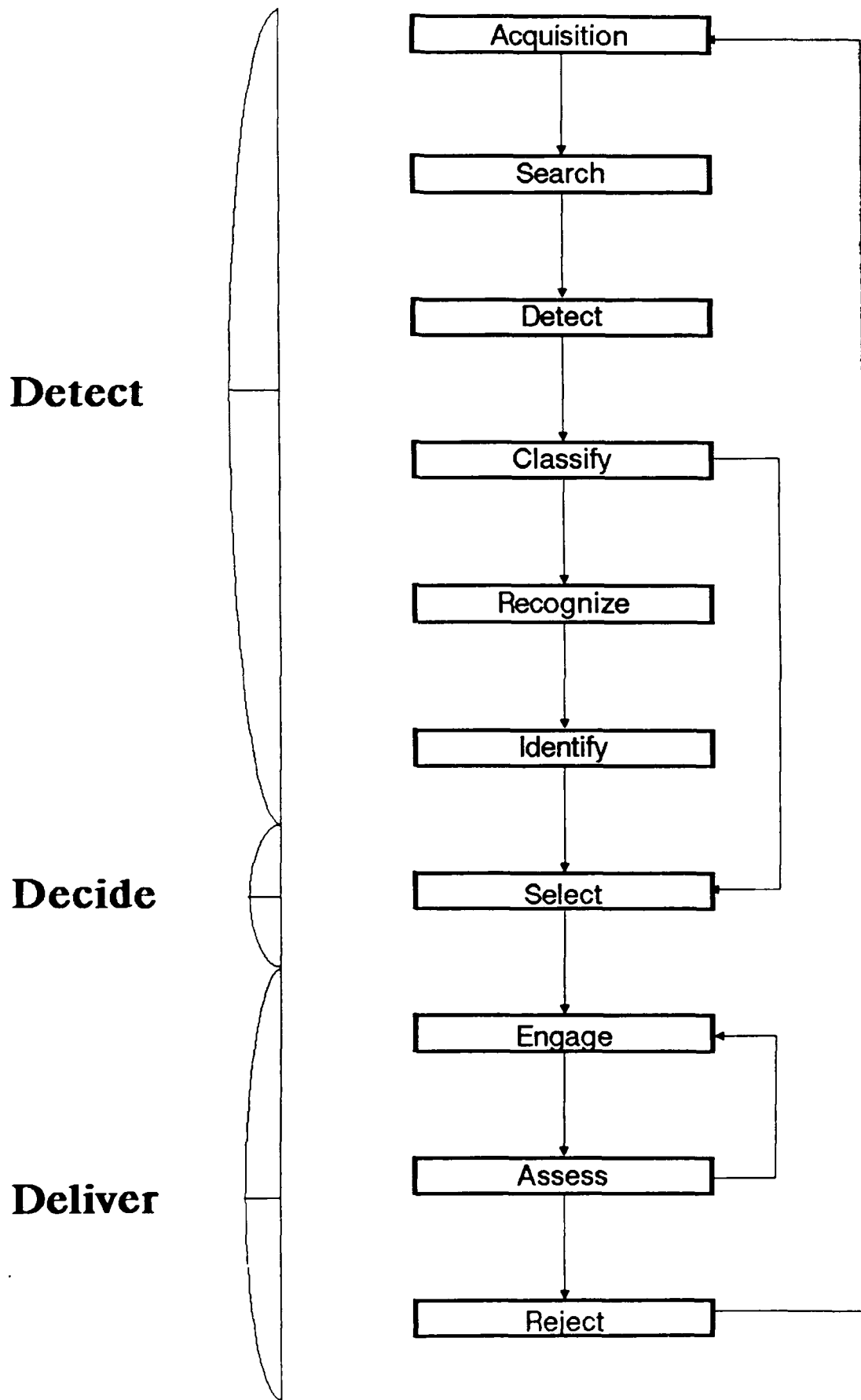
1. Detect
2. Decide
3. Deliver

#### ATTRITION PROCESS

This flow chart contrasts the conventional and smart munitions forms of the activities of the attrition process. The subactivities of the conventional form have been inserted.



# Attrition Process



DQ

### SERIAL vs. PARALLEL PROCESS

There are two ways that the attrition process activities may occur, either in series or in parallel. If an activity (or activities) occurs in series, then each activity is completed before another begins. If two activities occur simultaneously, then they occur in parallel.

# Serial vs Parallel Process

Serial - Perform one operation at a time.

Parallel - Perform several operations simultaneously (may be same operation repeatedly, different operations, or mixture.)

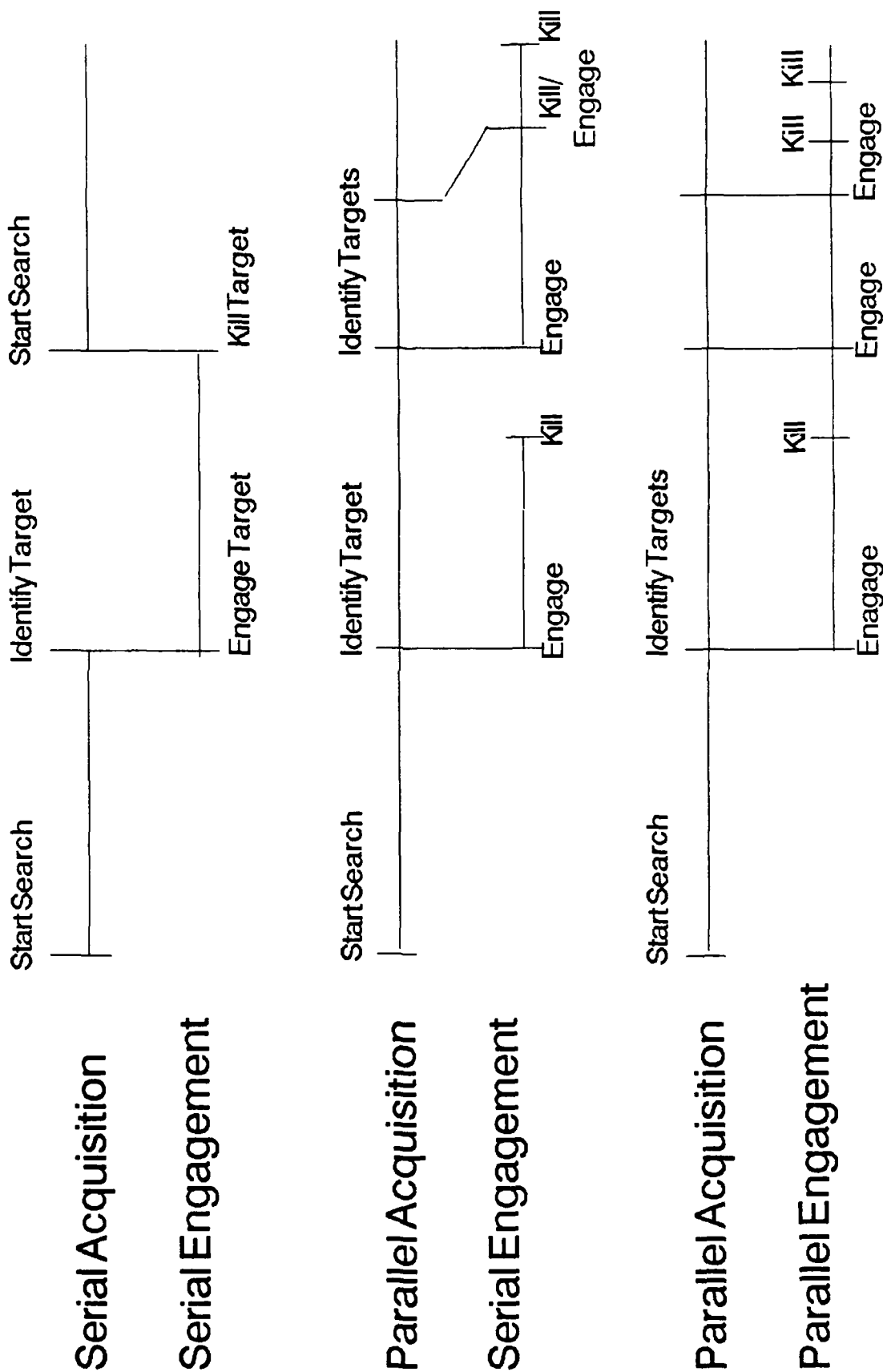
## SERIAL vs. PARALLEL ATTRITION

We depict the time lines associated with the two primary activity sequences of attrition: target acquisition and target engagement. If these sequences are serial, a target must be acquired before it can be engaged; once the target has been successfully engaged (target rejection has occurred), target acquisition can begin again. This is the slowest of the ways the process may occur. This type of attrition occurs when targets are hard to acquire and the acquirer does the engagement. An example of this could be an individual infantryman.

The second example depicts parallel acquisition with serial engagement. Once a target is acquired, it can be engaged as soon as the weapon is available and acquisition of another target can begin immediately. A simple form of this can be found in tank attrition when the commander acquires a target and directs the gunner to engage the target. Once the gunner has found the target, the commander can begin acquiring another target. Generally, this form of the attrition process is faster than the previous example.

The third example allows acquisition and engagement to occur simultaneously. An example of this would be a weapon with automatic target acquisition and automatic target engagement, with targets being tracked once they are acquired. This attrition process form is faster than the other two.

# Serial vs Parallel Attrition



#### TARGET ACQUISITION EVENTS

Six events may be associated with the target acquisition activity for visual acquisition. Each event can be related to physical parameters associated with the target and the acquisition device. There is some evidence that once combat is joined, the last three events may not always occur. This has the effect of speeding up the acquisition activity.

# Target Acquisition Events

1. Search (Look for Target.)
  2. See (Does LOS exist?)
  3. Detect (Notice the Target!)
  4. Classify (The Target is a vehicle or military object.)
  5. Recognize (The Target is a Tank.)
  6. Identify (The Target is an enemy tank of a particular model.)
- (nb Process may end between events 3 and 4.)

## TARGET ACQUISITION

Alternate definitions of three of the target acquisition events exist.



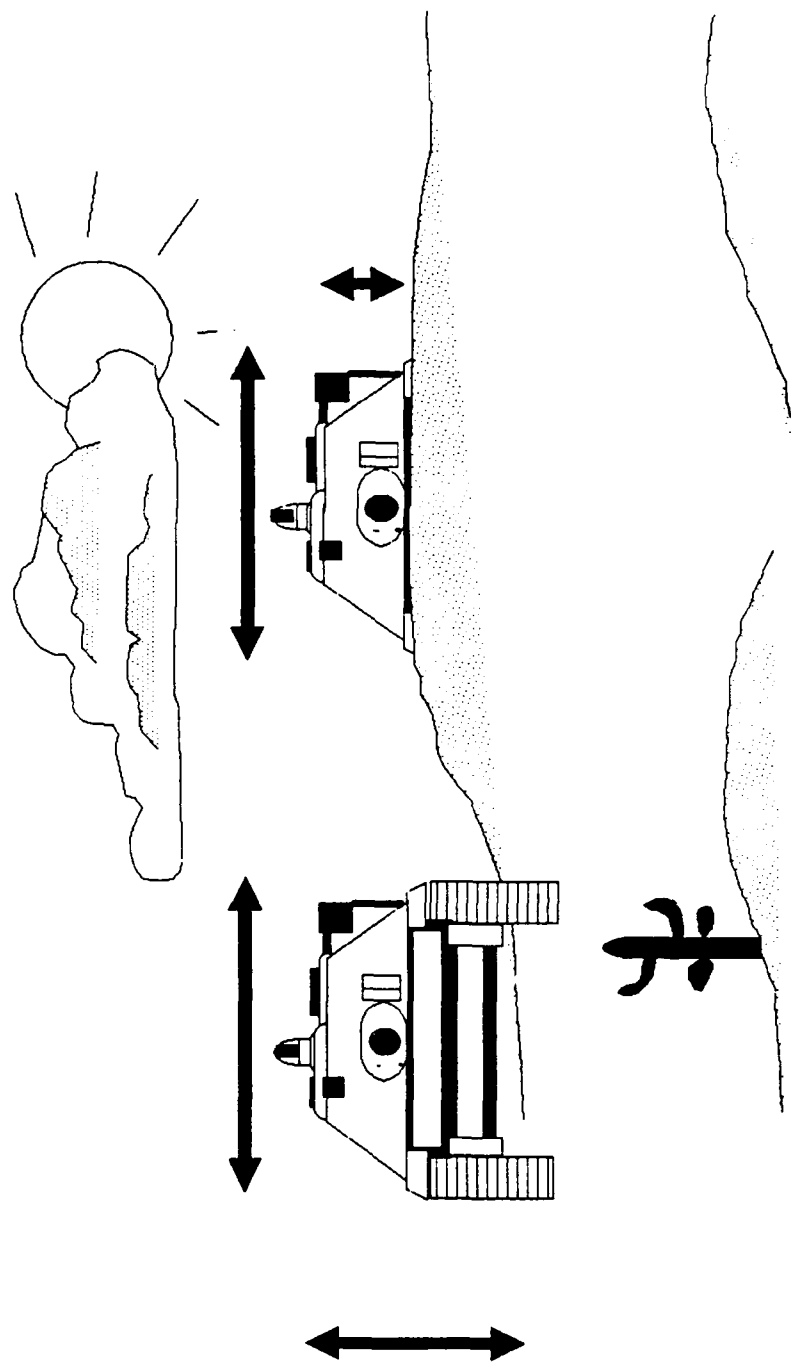
# Target Acquisition

1. Detection – the observer becomes aware of a local variation in the image structure.
2. Recognition – the observer sorts the variations into classes.
3. Identification – the observer specifies the variation to be a particular one of a class.

#### HULL DEFILADE EFFECTS

In both target acquisition and target engagement, the size of the target is a factor in the overall attrition process. This chart depicts the effects associated with a hull defilade tank as compared to a fully exposed tank. The latter presents a larger area. As we shall see, this area difference has a marked effect on both acquisition and engagement.

# Hull Defilade Effects



Fully Exposed Tank    Hull Defilade Tank

DP

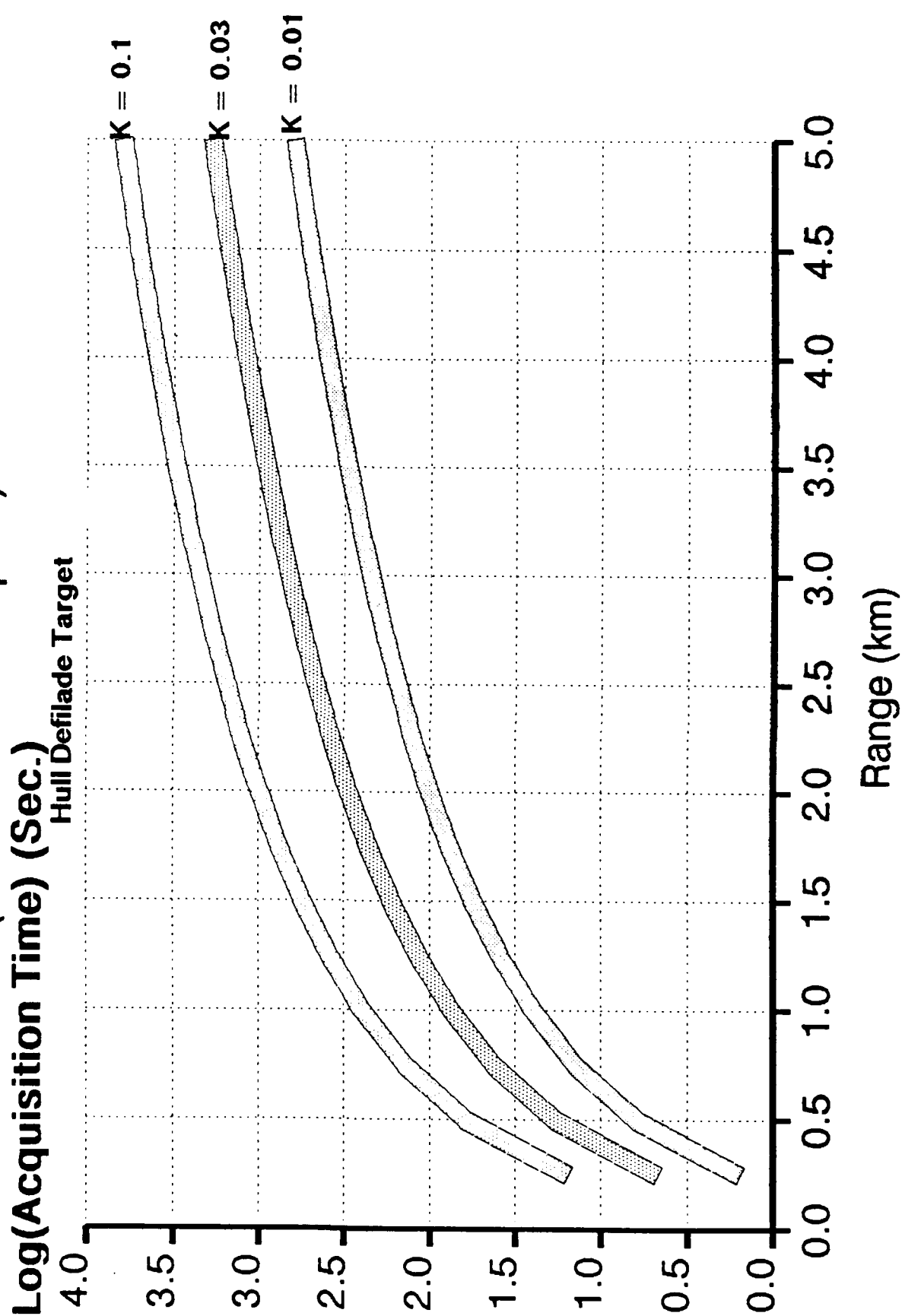
#### ACQUISITION TIME

Random Glimpse;  
Hull Defilade Target

This plot presents three curves of acquisition time versus range.

The curves vary with a scene clutter parameter K. The larger K is, the greater the scene clutter.

# Acquisition Time (Random Glimpse)

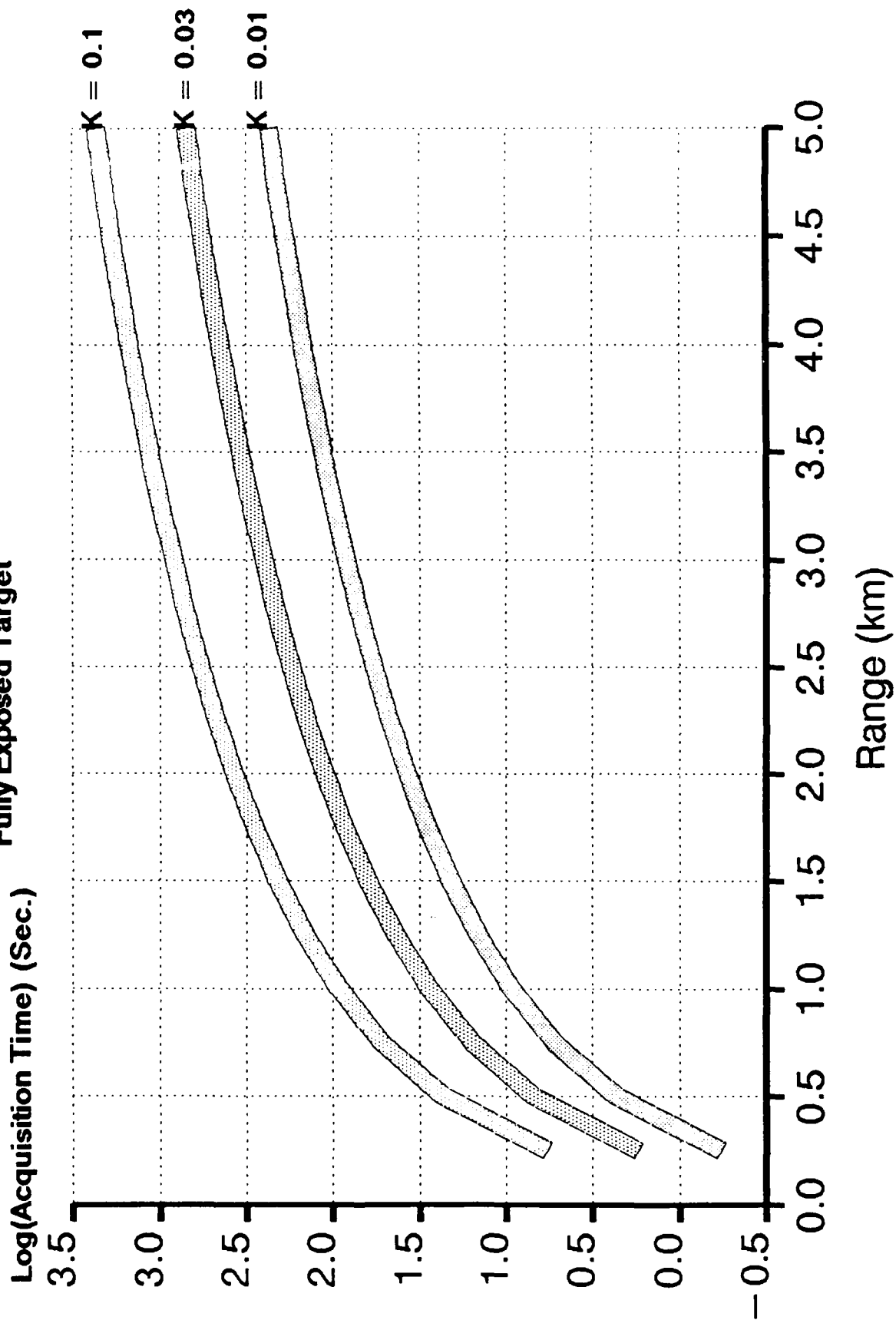


#### ACQUISITION TIME

Random Glimpse;  
Fully Exposed Target

The same types of curves as the previous chart, except for a fully exposed tank, may be plotted. Comparison of the two curves shows the longer acquisition times required for the hull defilade tank.

# Acquisition Time (Random Glimpse) Fully Exposed Target



## PROBABILITY THAT OBJECT IS IN SEARCH AREA

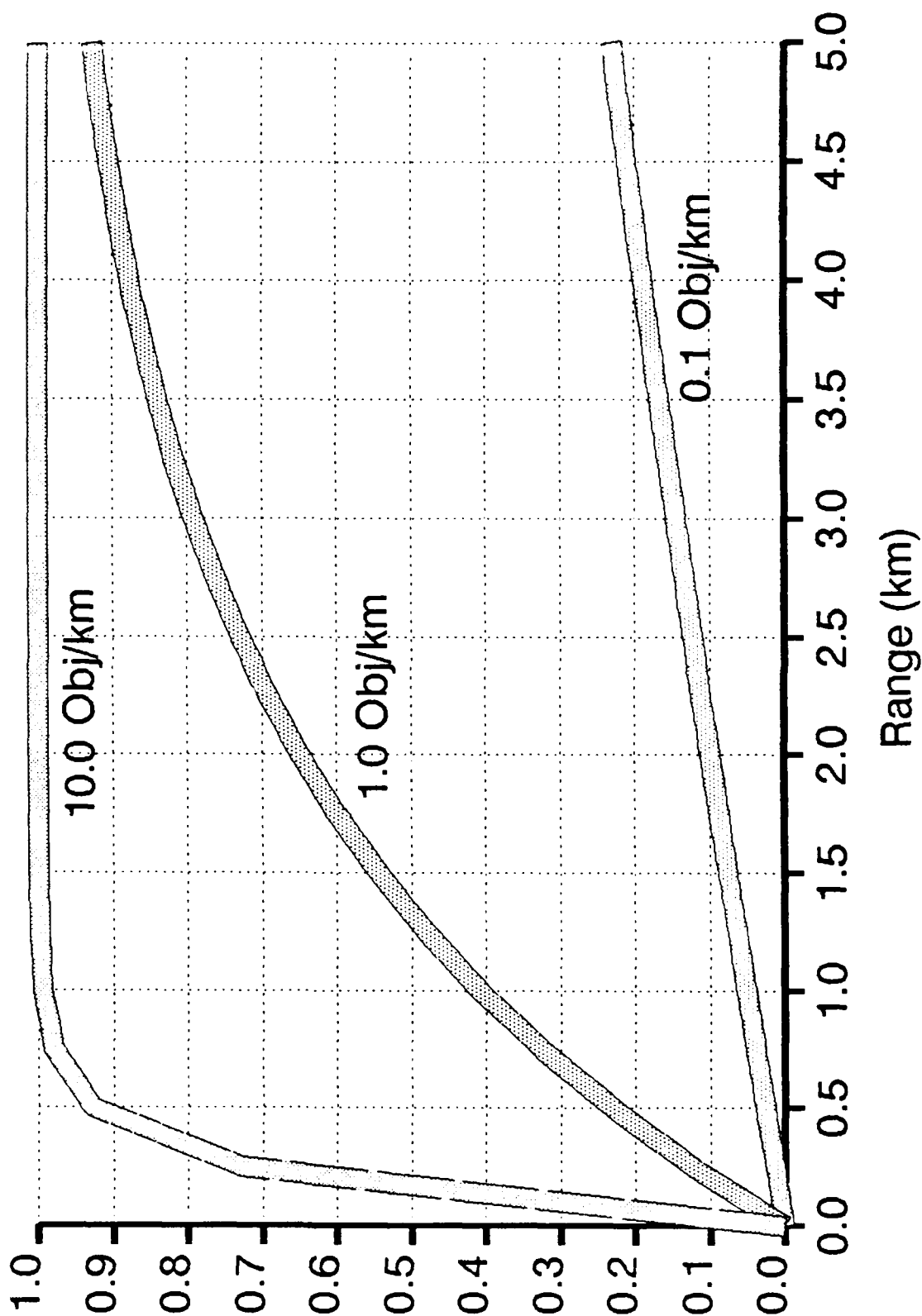
### Linear Formation

This plot shows the probability that an object is in an area being searched given some density of targets. In this case, the targets are assumed to be in a linear array which is normal to the observer (the targets approach in line abreast). As the array becomes less dense, the chance of finding the target decreases.



# Probability that Object in Search Area

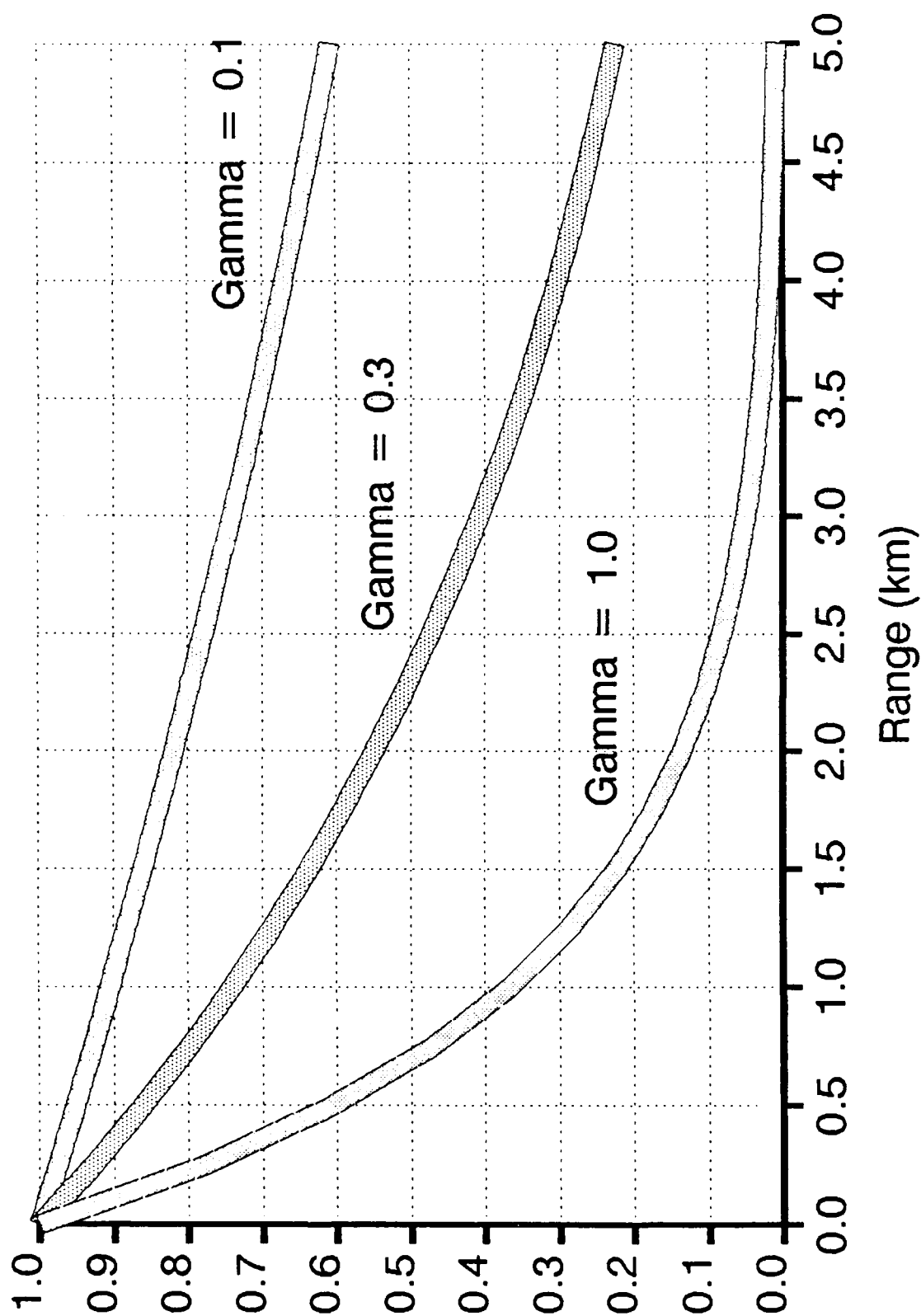
## Linear Formation



#### PROBABILITY OF A CLEAR LOS

This plot presents the probability of a clear line-of-sight between observer and target versus range for different values of a terrain roughness parameter. As the terrain gets rougher, the chance of an LOS occurring becomes less.

# Probability of a Clear LOS



### NORMAL PROBABILITY DISTRIBUTION

Probability of detection (recognition and identification) is (are) usually assumed to be normally distributed. The first equation is for the normal or Gaussian probability distribution.

The second equation is an approximation of the normal probability distribution (the integrals are hard to perform) when the sight point is the center of the target (in terms of presented area). This approximation has an accuracy of about 0.01%. The  $X$  and  $Y$  variables have values equal to the half height and half width (in length units) of the target area; the standard deviations, designated by the greek letter sigma, represent the probability distribution bounds on acquiring a target of that height/width.

# Normal Probability Distribution

$$p[ ] = \frac{1}{2 \pi} \int \int e^{-X^2/2\sigma_X^2} e^{-Y^2/2\sigma_Y^2} dx dy$$

Total  
Area

## Centered Target Approximation

$$p[ ] = \sqrt{(1 - e^{-2X^2/\pi\sigma_X^2})(1 - e^{-2Y^2/\pi\sigma_Y^2})}$$

#### TARGET SIGNAL/NOISE

The ratio of target height or width to standard deviation is related to a quantity called the signal-to-noise ratio (actually the square of that ratio). These parameters depend on the weather, the target, and the nature of the acquisition device (sensor). These two equations represent signal-to-noise ratio in the visible (eyeball) and far infrared (FLIR) spectral regions. Both apparent contrast and resolvable cycles depend on the atmospheric transmittance, which is a function of the weather.

# Target Signal/Noise

## Visual Spectral Band

$$S_n \approx \frac{1}{0.39} \left[ \frac{C}{C_t} - 1 \right]$$

## Far InfraRed Spectral Band

$$S_n \approx \frac{N[s] - \bar{N}}{\sigma}$$

$C_t$  = Threshold Contrast

$C$  = Apparent Contrast

= Inherent Contrast \* Transmittance

$N[s]$  = number of resolvable cycles

$\bar{N}$  = Expected number of resolvable cycles

$\sigma$  = standard deviation of number of resolvable cycles

CZ

### THRESHOLD CONTRAST

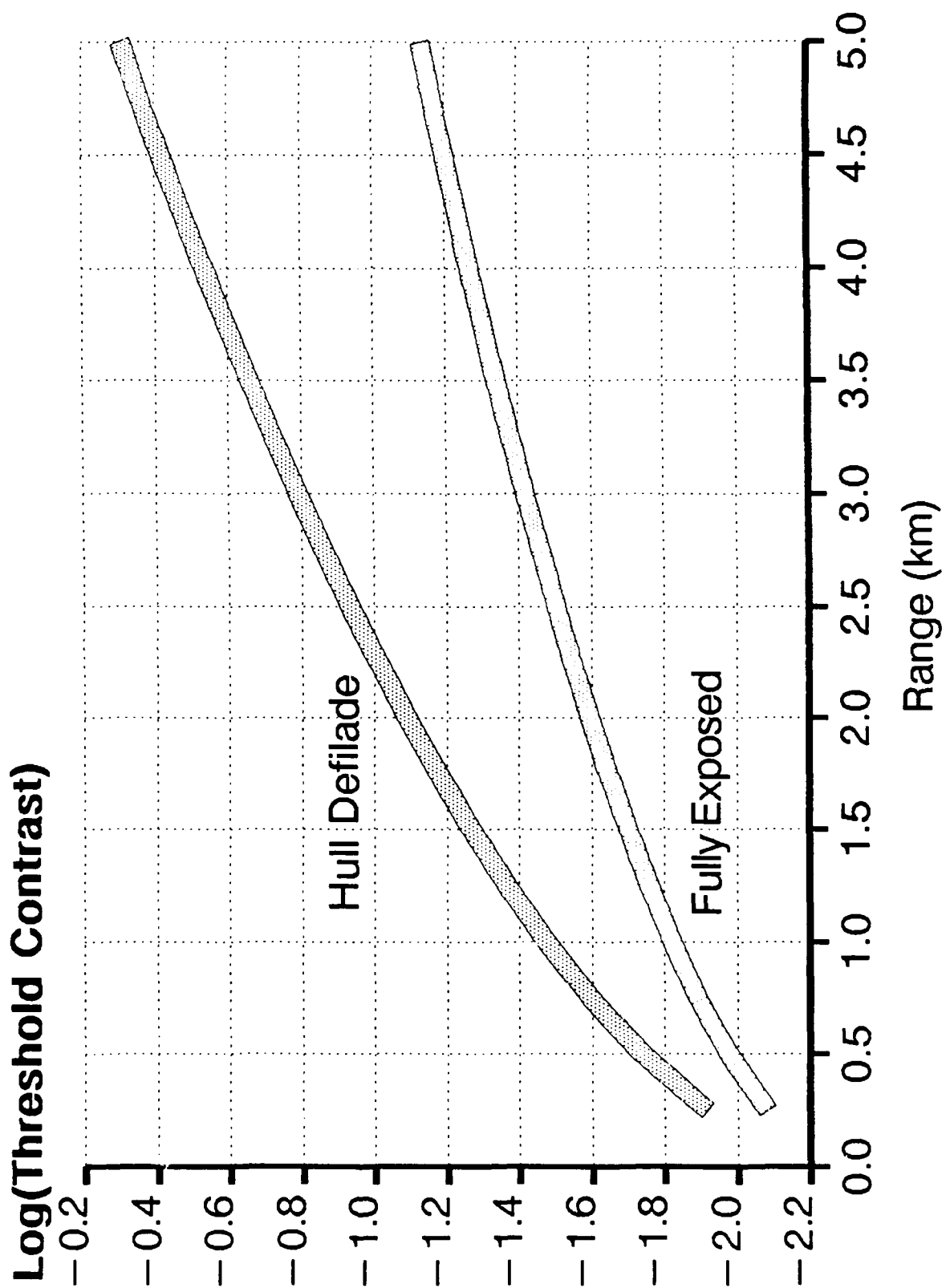
Inherent Contrast = 0.2

This plot presents values of the threshold contrast plotted versus range. The threshold contrast is a parameter used in calculating the signal-to-noise ratio for the visible spectral region (introduced in the previous chart). The two types of target, hull defilade and fully exposed, are shown. Notice that much larger threshold contrasts are required for the hull defilade target than for the fully exposed target. This is a direct consequence of the presented area of the two types of targets.



# Threshold Contrast

Inherent Contrast = 0.2

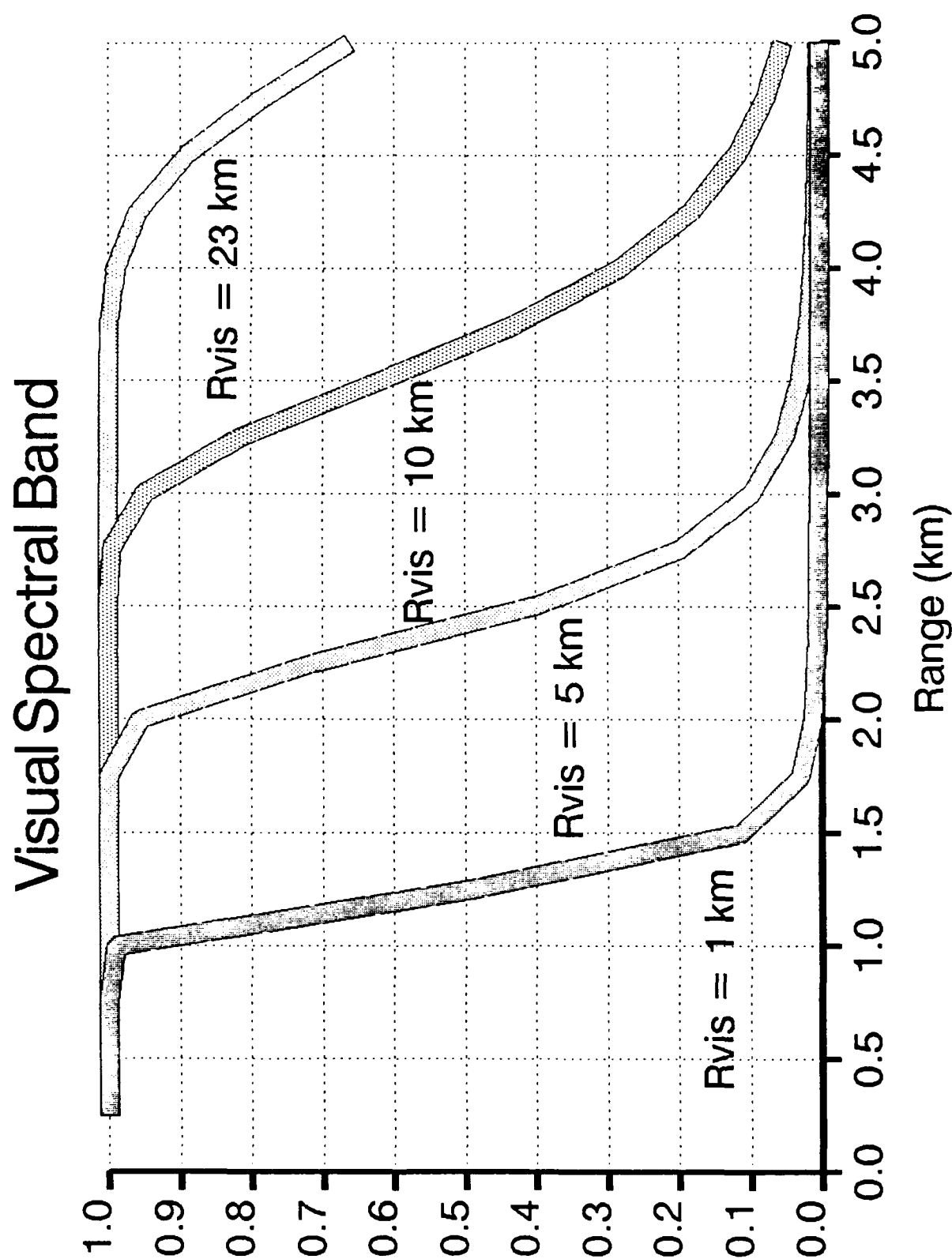


# PROBABILITY OF DETECTION FULLY EXPOSED

## Visual Spectral Band

Curves of the probability of detection of the fully exposed target versus range for several different visibility ranges are presented. A visibility range of 23 km represents a very clear day, while a visibility range of 1 km represents fog.

# Probability of Detection Fully Exposed



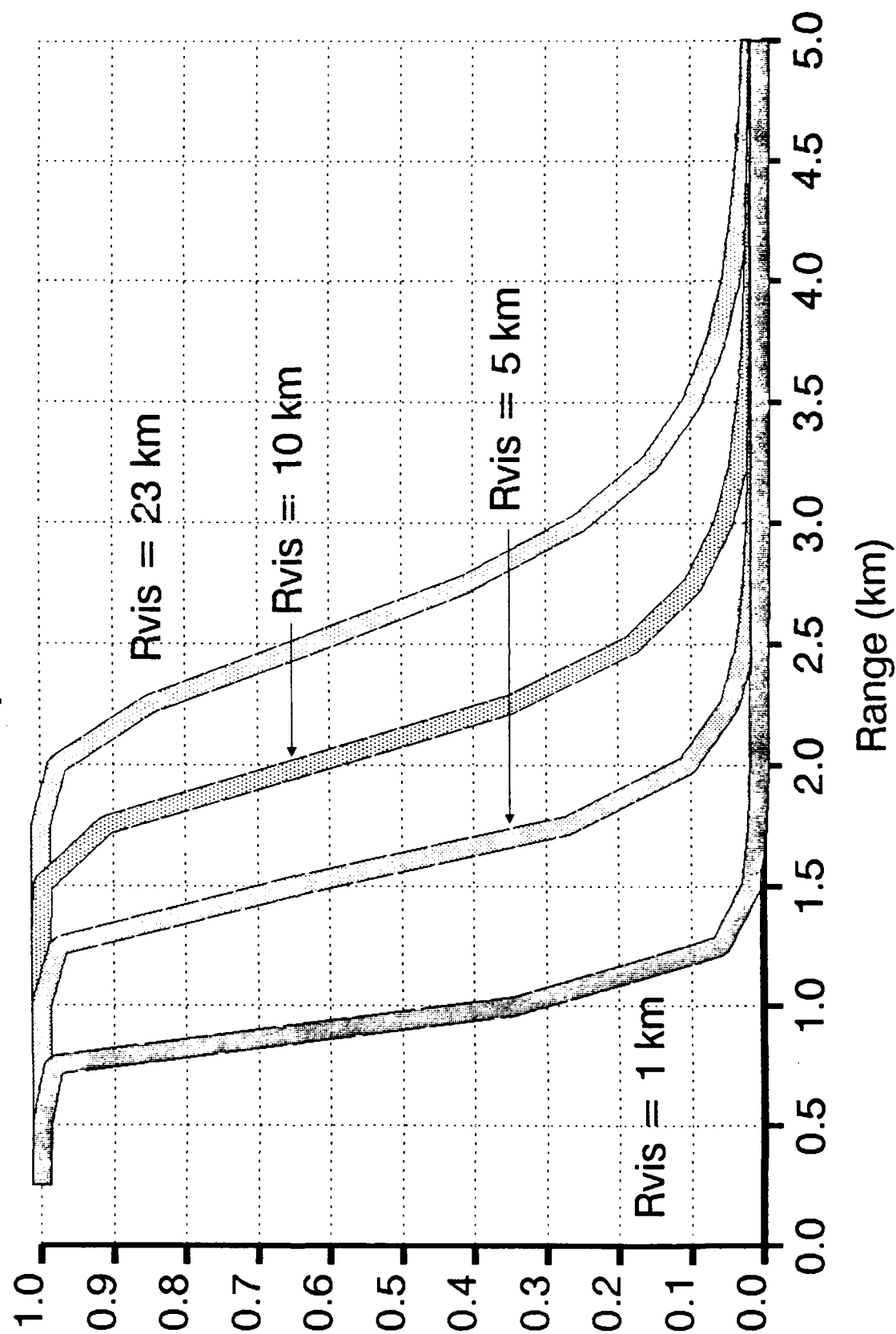
## PROBABILITY OF DETECTION HULL DEFILADE

### Visual Spectral Band

This plot presents the same type of information as the preceding chart except for the hull defilade target. Note that the hull defilade target is harder to detect, even on a clear day. This is again a direct consequence of the difference in the presented areas of the two types of target.

# Probability of Detection Hull Defilade

Visual Spectral Band



## TARGET ENGAGEMENT PROCESS

The target engagement process, which normally follows the target acquisition process, consists of three phases: (1) a target sighting phase, when the weapon is brought to bear on the target as it is tracked, and an aim point is selected; (2) a firing phase, when the weapon is discharged at the target (this phase includes the flight of the shell or missile up to the target, and may include the actual impact); and (3) a kill assessment phase, during which the firing unit (weapon system operator) determines whether the target has been killed. If doctrine does not dictate otherwise, and the weapon does not run out of rounds, then this process cycles until the target is killed. If track is maintained on the target during the fire and assessment phases, the sighting phase does not need to be repeated.

# Target Engagement Process

1. Sight Target
2. Fire at Target
3. Perform Kill Assessment.

## ENGAGEMENT PROCESS

### Times and Probabilities

The engagement process has both time and probabilities associated with it. For example, the engagement time is the time for one round to be fired and assessment performed. For a modern tank, the engagement time has been found historically to be approximately the inverse of the sustained rate of fire. This compact form of the engagement time is due to the high speed of the modern tank round (800-1600 m/sec), and the demonstrative nature of tank kill (the "brew-up").

Being slower, the second-generation ATGM has a somewhat different form of the engagement time. Because the most common launchers, such as TOW or MILAN, are single-shot devices, a reload time must be included. The time of flight to range is also typically a factor since most ATGM speeds are 5-10 times slower than tank rounds. Finally, an assessment time must be included.

Associated with each engagement are two probabilities: the probability of hit given a shot, and the probability of kill given a hit. The product of these two gives the probability of kill given a shot. In actuality, probability of kill given a shot cannot usually be calculated independently, since the likelihood of kill depends on where the round hits the target. Thus, most frequently, probability of hit given a shot and probability of kill given a shot are calculated, and probability of kill given a hit is back-calculated from these two.

The hit probability in this case is taken to have a Gaussian (or normal) probability distribution over a rectangular target. Tank silhouettes are approximately rectangular. The kill probability is approximated by a constant. If the aim point can be well defined, and the standard deviations of shot error are small, then this approximation is highly valid.



# Engagement Process

## Times and Propabilities

### Engagement Time

$$t[\text{Engagement: Tank}] \sim \frac{1}{(\text{rate of fire})}$$

$$t[\text{Engagement: ATGM}] \sim t[\text{Reload}] + \frac{r}{v_{\text{msl}}} + t[\text{Evaluation}]$$

### Hit Probability

$p[\text{Hit}] \sim \text{Normal Probability}(\text{Rectangular Target})$

### Kill Probability

$$p[\text{Kill} \mid \text{Hit}] \sim \text{Constant}$$

### TARGET SELECTION, TARGET REJECTION

In some instances, two other attrition processes are included: target selection and target rejection. Fundamentally, they represent the transitions from acquisition to engagement, and from engagement to acquisition, respectively. Note that in target selection, the attrition process moves from acquiring a target to engaging the same target; in target rejection, the attrition process moves from engaging a target to acquiring another target. In the example of direct fire, target selection is the process by which the target acquirer (e.g., tank commander) tells the target engager (e.g., tank gunner) where the target is; target rejection is the process of the target engager telling the target acquirer that the previous target is killed and that a new target is needed. These two processes also have times associated with them which modify the attrition rate.

# **Target Selection**

**The Process of changing from  
Acquisition to Engagement.**

# **Target Rejection**

**The Process of changing from  
Engagement to Acquisition.**

### BONDER'S EQUATION

The fundamental equation describing the attrition rate for Square Law, direct fire processes is Bonder's Equation, named after Seth Bonder, who developed this equation as part of his doctoral research. This equation says that the attrition rate--the rate at which targets are killed by a single unit--is the inverse of the expectation value (average) of the total attrition time. The expectation value is calculated using the times and probabilities associated with the phases of the attrition process which were described in the preceding charts.

## Bonder's Equation

$$\text{Attrition Rate} = 1/E(T)$$

where:  $E(T)$  is the expectation value of the attrition time.

#### ATTRITION TIME

In terms of the processes previously described, and neglecting target selection and rejection, the attrition time is, in the conventional form, the sum of the acquisition and engagement times. In the smart munition form, the attrition time is the sum of the detection, decision, and destruction times.

# Attrition Time

## Conventional Form

$$T[\text{attrition}] = T[\text{acquisition}] + T[\text{engagement}]$$

## Smart Munition Form

$$T[\text{attrition}] = T[\text{detection}] + T[\text{decision}] + T[\text{destruction}]$$

## ACQUISITION FACTORS

Six factors make up the acquisition process:

- (1) The presence of the target in the area (or solid angle) being searched. This is a probabilistic factor.
- (2) Is the target looked at? This is a time factor and a probability of the time it takes for the searcher to look at the target.
- (3) Is the target detected? This is a probability factor.

The other three factors-- (4) classification, (5) recognition, and (6) identification-- occur once the target has been detected. They have probability factors associated with them. For visual acquisition, they have the same form as but different parameters from detection. There is some historical evidence that once combat is joined, these three factors do not enter into the engagement process--that is, once something is detected that may be an enemy, it is acquired and fired on.



# Acquisition Factors

1. Target is in search field?
2. Target is Looked at?
3. Target is Detected?
4. Target is Classified?\*
5. Target is Recognized?\*
6. Target is Identified?\*

#### ENGAGEMENT FACTORS

The factors associated with engagement are only two: shooting and kill assessment.

# Engagement Factors

1. Target is shot at?
2. Target is killed?

## EXPECTED TIMES

### Conventional Form

Because the attrition time is linear in acquisition and engagement times, and we treat these as independent in the simplest case of serial acquisition and engagement, the expected value of the attrition time is just the sum of the expected values of the acquisition and engagement times.

The expected value of the acquisition time is just the time for acquisition (basically the time required for the searcher to look at the target long enough for a detection to occur) divided by the probabilities that the target is in the search area (or solid angle), that line of sight exists to the target, and that detection occurs.

The expected value of the engagement time is just the time of engagement divided by the probability of kill per shot. In other words, the time to engage the target with one round (compensated for reload and other factors) times the average number of shots needed to achieve a kill.

# Expected Times

Conventional Form

## Expected Attrition Time

$$E(T[\text{attrition}]) = E(T[\text{acquisition}]) + E(T[\text{engagement}])$$

## Expected Acquisition Time

$$E(T[\text{acquisition}]) \sim t[\text{acquisition}] / \{ p[\text{Search}] p[\text{LOS}] p[\text{Detection}] \}$$

## Expected Engagement Time

$$E(T[\text{engagement}]) \sim t[\text{engagement}] / \{ p[\text{hit}] p[\text{kill}] p[\text{hit}] \}$$

#### **NORMAL PROBABILITY DISTRIBUTION**

#### **CENTERED TARGET APPROXIMATION**

This is a repeat of an earlier chart to reiterate the form of the normal (or Gaussian) probability distribution, which described both acquisition and engagement factors, and the centered target approximation, which assumes that the searcher looks at the center of the area of the target and the engager aims at the same point.

# Normal Probability Distribution

$$p[ \ ] = \frac{1}{2 \pi} \int \int e^{-X^2/2\sigma_x^2} e^{-Y^2/2\sigma_y^2} dX dY$$

Total  
Area

## Centered Target Approximation

$$p[ \ ] = \sqrt{(1 - e^{-2X^2/\pi\sigma_x^2})(1 - e^{-2Y^2/\pi\sigma_y^2})}$$

**SQUARE TARGET UNIFORM SIGMA  
APPROXIMATION**

**DISPERSION SIGMA**

If the target is square, and the vertical and horizontal dispersions (standard deviations of the probability distribution) are equal, then the square target approximation may be formed.

The dispersion sigmas, for tank guns at least, are commonly found to be approximately linear functions of the range to the target. The meanings of these dispersions are shown graphically in the following discussion.



# Square Target Uniform Sigma Approximation

$$p[\ ] = \sqrt{1 - e^{-A/2\pi\sigma^2}}$$

## Dispersion Sigma

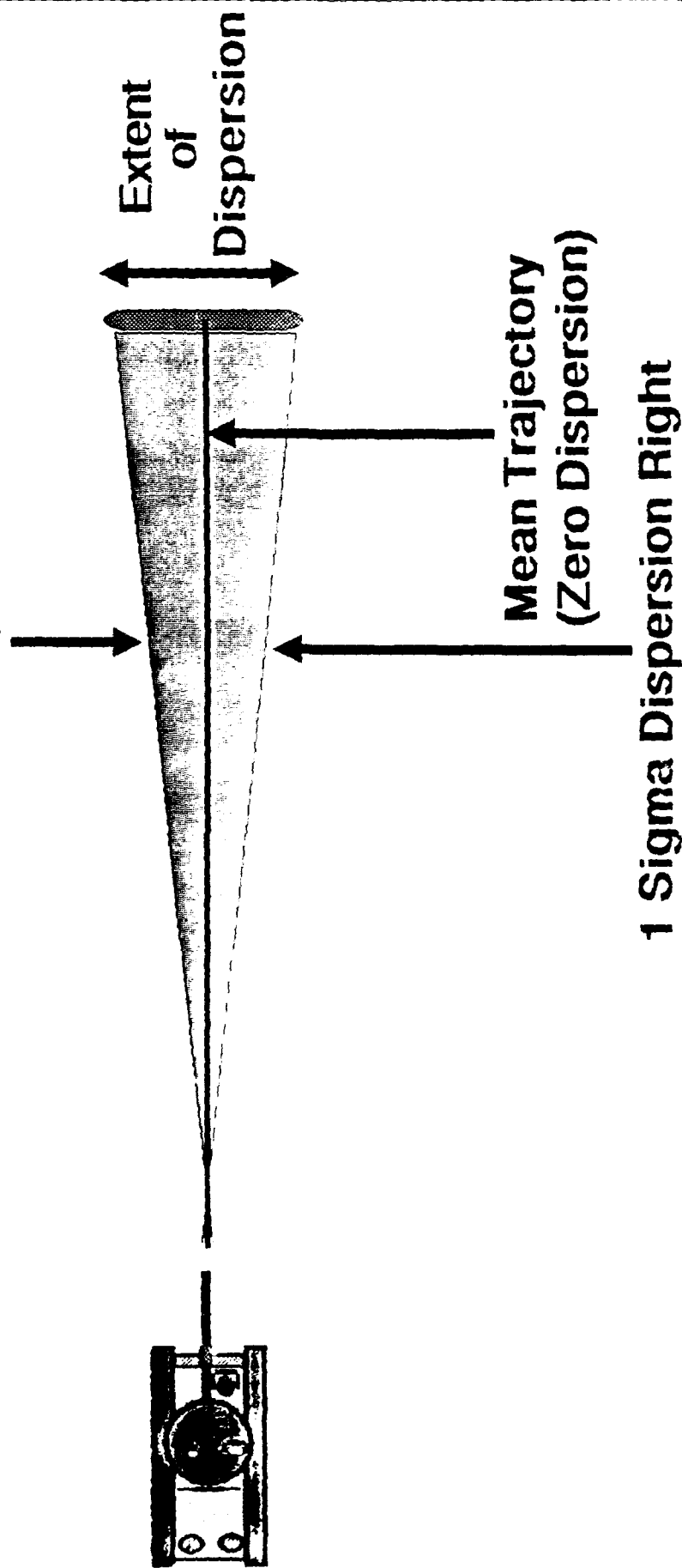
$$\sigma \propto R$$

## HORIZONTAL DISPERSION

The dispersion effects associated with the horizontal dispersion are shown. The tank is firing down range. The mean trajectory is shown in the heavy line. This trajectory is normally on the line of sight to the target. The one-sigma bounds on the trajectory are shown in the shaded area. As long as the target's dimension is less than the three-sigma bound, a hit is virtually assured. This picture ignores any outside influences, such as wind, which cause an asymmetry of the dispersion sigmas--the probability distribution shifts with the wind. Wind effects are small for high-speed or guided projectiles such as modern tank rounds and ATGMs. Modern fire control systems compensate for wind effects and usually incorporate a wind speed sensor and a range finder, since both the wind speed and the range to the target must be known.

# Horizontal Dispersion

1 Sigma Dispersion Left



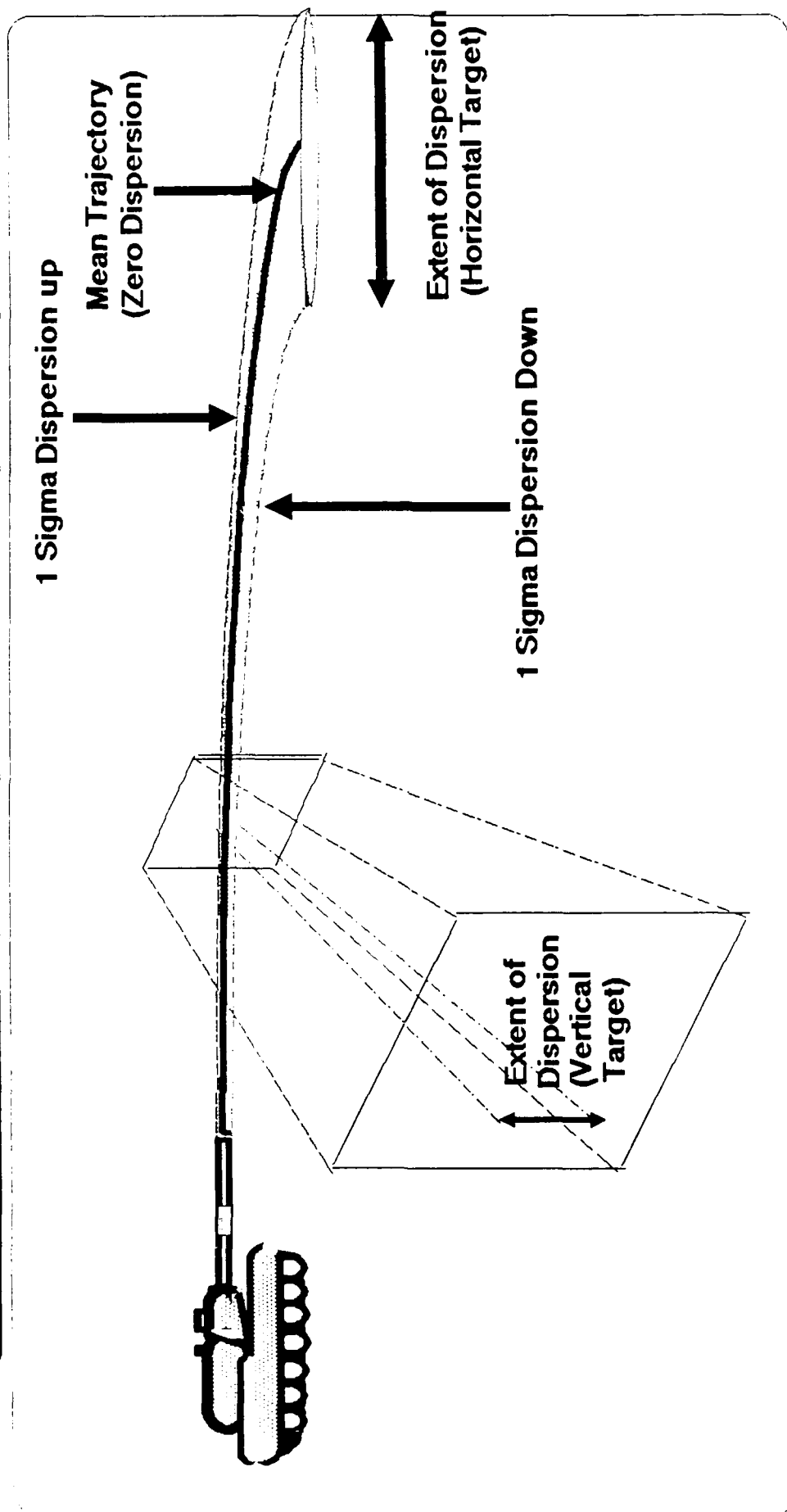
At long range, can ignore arrival angle effects, must include wind and target motion effects.

DK

### VERTICAL DISPERSION

Vertical dispersion for a direct fire engagement is shown. Note that the dispersion pattern is skewed at long range (near the ground impact point) due to gravity. Modern fire control systems compensate for gravity so as to reduce this skew. The picture does not include these corrections, which are dependent on knowing the range to the target. Additionally, the impact range of high-speed projectiles is frequently much larger than engagement ranges, so gravity effects are minimized.

# Vertical Dispersion



**For high projectile speed, moderate range, etc, can ignore arrival angle effects for vertical targets (i.e. direct fire.) Gravity effects can be included in dispersion.**

### HIGH ANGLE VERTICAL DISPERSION

This shows vertical dispersion for an indirect fire engagement. (There is little difference in horizontal dispersion for direct and indirect fire.) Note that not only is the dispersion skewed, but the roles of the upper and lower sigmas are swapped in the impact area. The down sigma trajectories travel farther than the mean trajectory, and the up sigma trajectories travel a smaller distance than the mean trajectory. The indirect fire case is complicated by the fact that the range to the target is usually not well known, since line of sight does not usually exist.

Further, wind effects can also skew the dispersion pattern by lengthening or shortening the range to impact. Thus wind normally affects only horizontal dispersion in the direct fire case, but both horizontal and vertical dispersion in the indirect fire case.

# High Angle Vertical Dispersion

1 Sigma Dispersion Up



Mean Trajectory

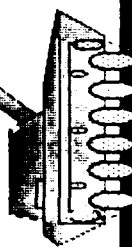
(Zero Dispersion)



1 Sigma Dispersion Down



Extent of Dispersion



## ACCURACY EFFECTS

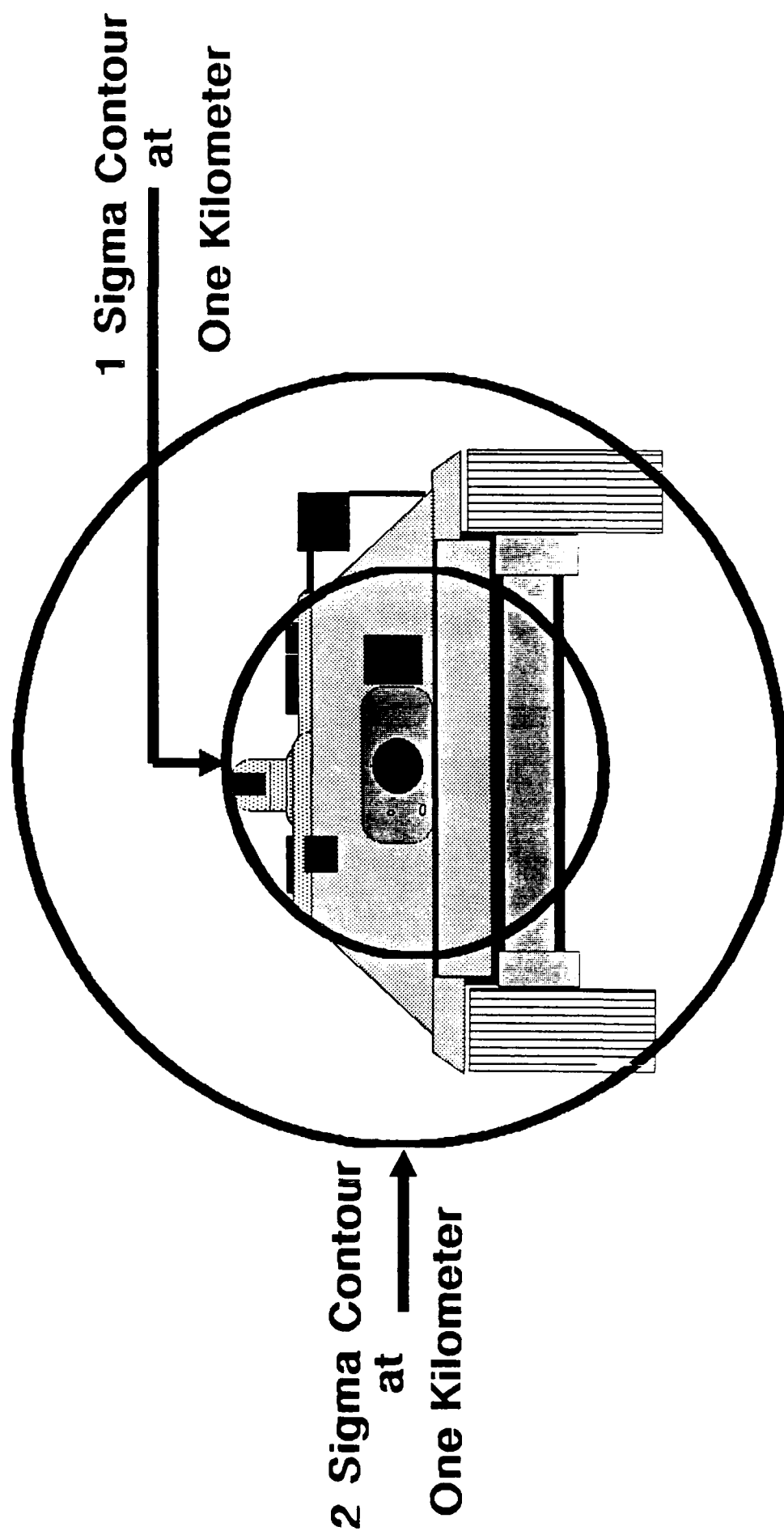
### *Aim Point on Center*

The concentric circles show the effect of sigma accuracy. As the dispersion area increases, less of the target is in that area, so the probability of hit decreases.



# Accuracy Effects

Aim Point on Center



$P[\text{kill}; \text{hit}]$  is a non-independent quantity calculated from  $p[\text{kill}; \text{shot}]$  and  $p[\text{hit}; \text{shot}]$ .

DN

## ACCURACY EFFECTS

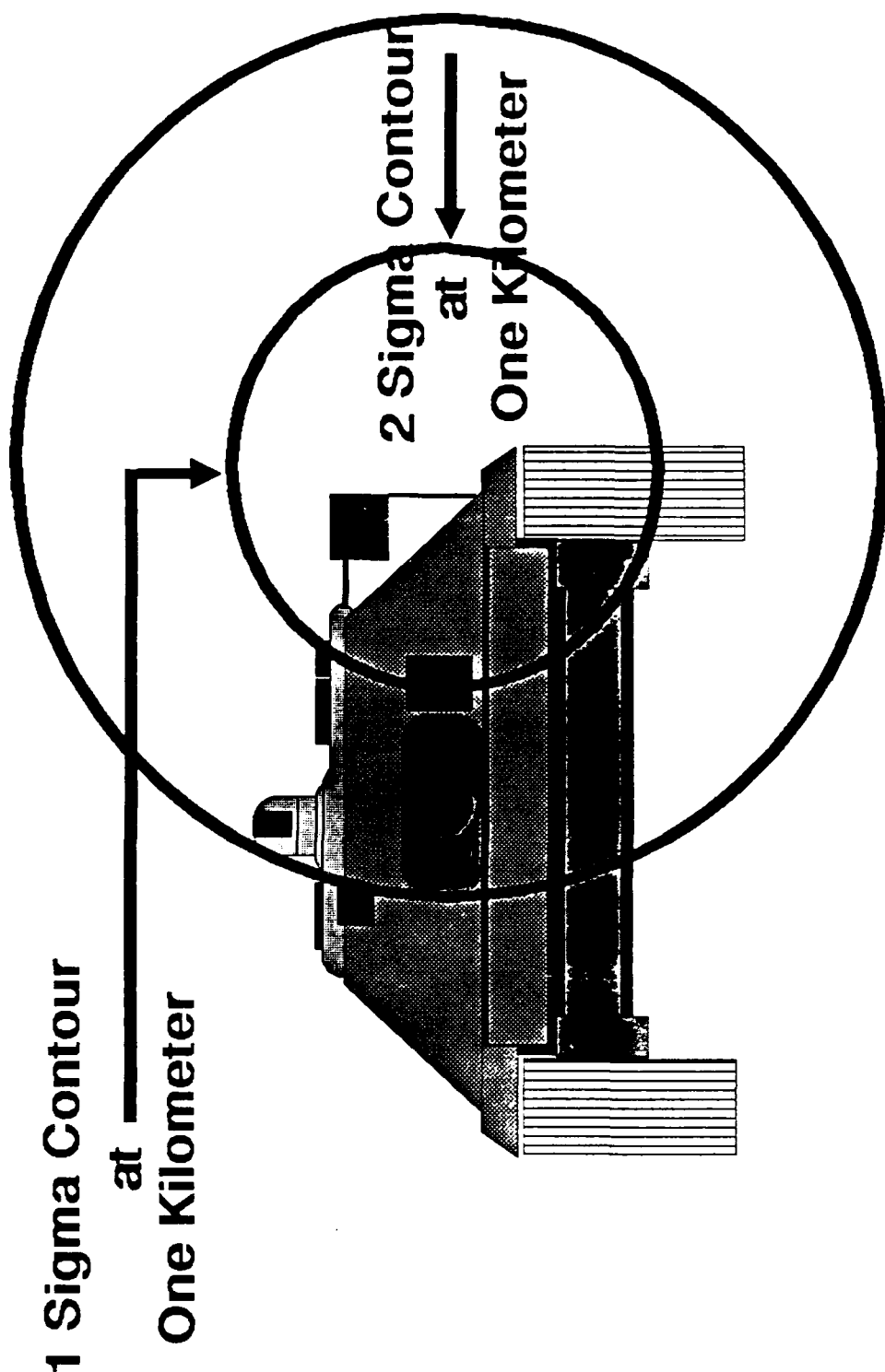
### Aim Point Effects

As the aim point differs more from the center of the target, less target area lies in the dispersion pattern, and there is less chance of hitting the target.

This type of situation can commonly occur at long range with strong wind, which pushes the dispersion area away from the line of sight aim point.

# Accuracy Effects

## Aim Point Effects



**$P[\text{kill} | \text{hit}]$  is a non-independent quantity calculated from  $p[\text{kill} | \text{shot}]$  and  $p[\text{hit} | \text{shot}]$ .**

DO

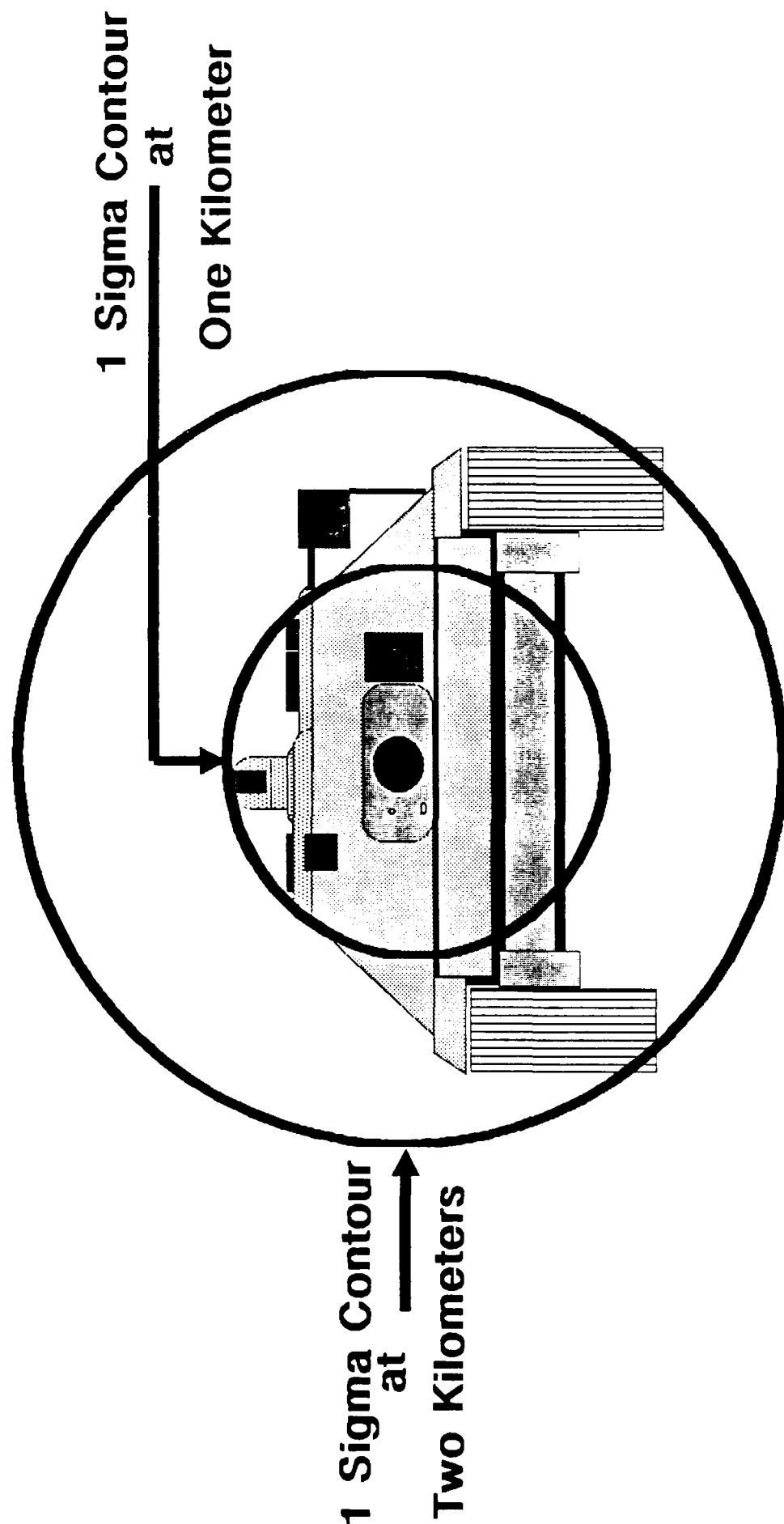
## ACCURACY EFFECTS

### Range Effects

This shows the effect of range on accuracy. As range doubles, the area of the dispersion increases by a factor of four. Targets thus become harder to hit as range increases.

# Accuracy Effects

## Range Effects



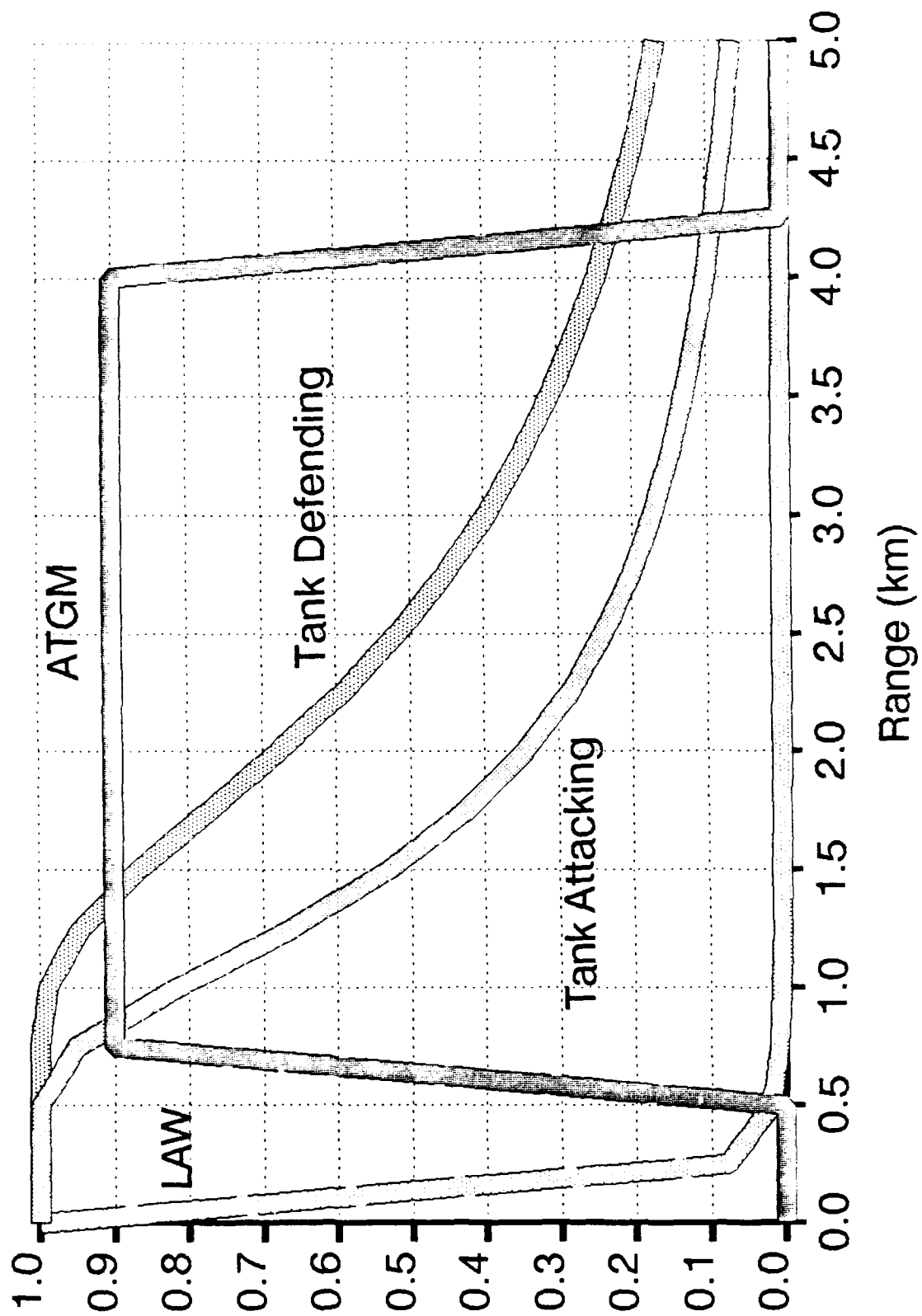
$P[\text{kill}; \text{hit}]$  is a non – independent quantity calculated from  $p[\text{kill}; \text{shot}]$  and  $p[\text{hit}; \text{shot}]$ .

DM

### PROBABILITY OF HIT

This graph shows probability of hit versus range for four different weapons: the Light Antitank Weapon, a shoulder-fired rocket; tanks attacking and defending. The defending tank shoots at a fully exposed target, the attacking tank shoots at a hull defilade target. The differences in the two curves are due to the differences in visible areas of the two targets; and an ATGM. Because of its high accuracy, the differences in visible area of the two types of targets are minor and not shown. This type of behavior is common in second-generation ATGMs.

# Probability of Hit

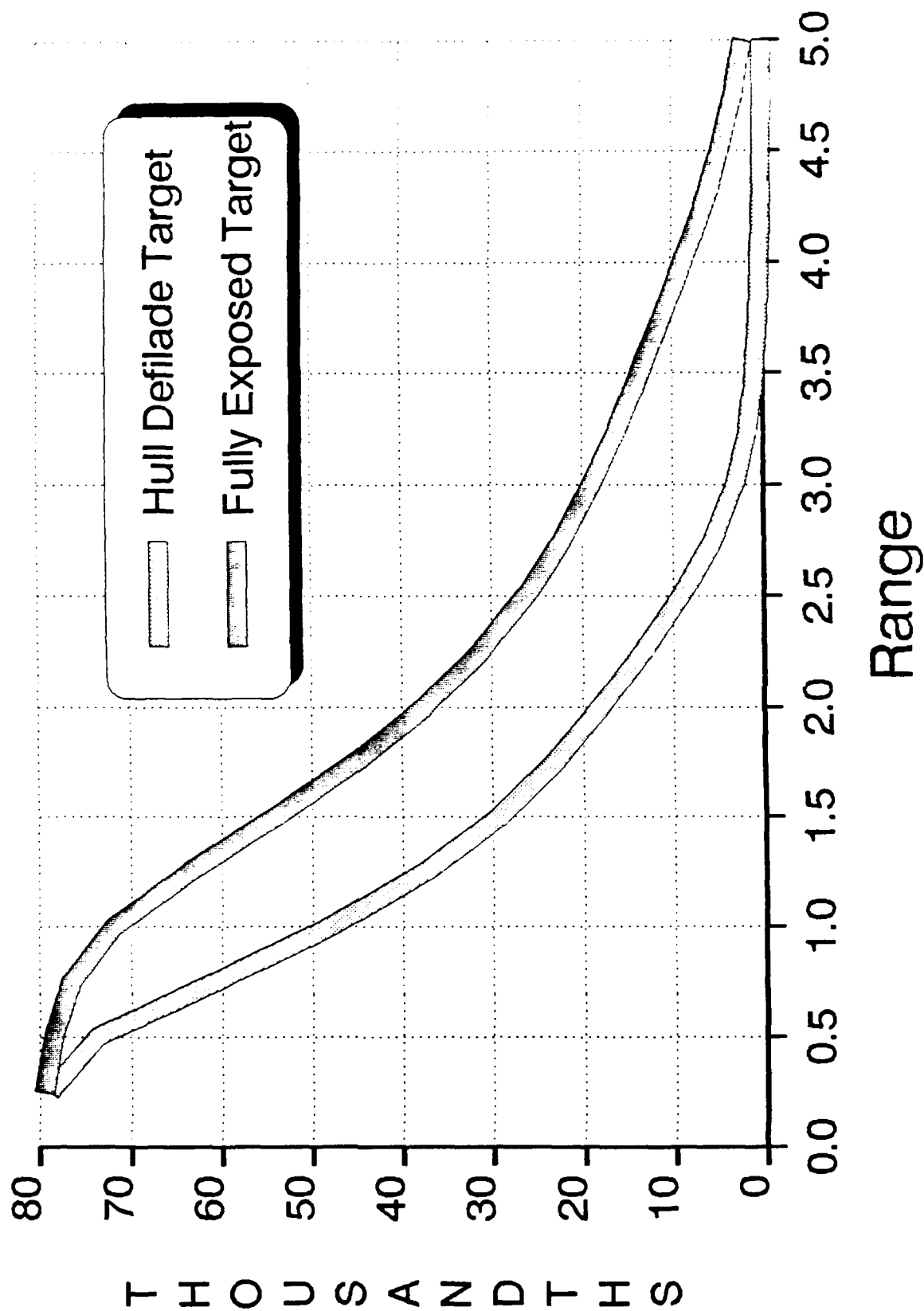


#### RANGE VARIABLE ATTRITION RATE

If we combine all of the factors, times, and probabilities in this portion of the report, we may use Bonder's equation to calculate range-dependent attrition rates for defending and attacking forces--for hull defilade and fully exposed targets. Note that the defender has an advantage here in the rate that he can kill attackers.



# Range Variable Attrition Rate



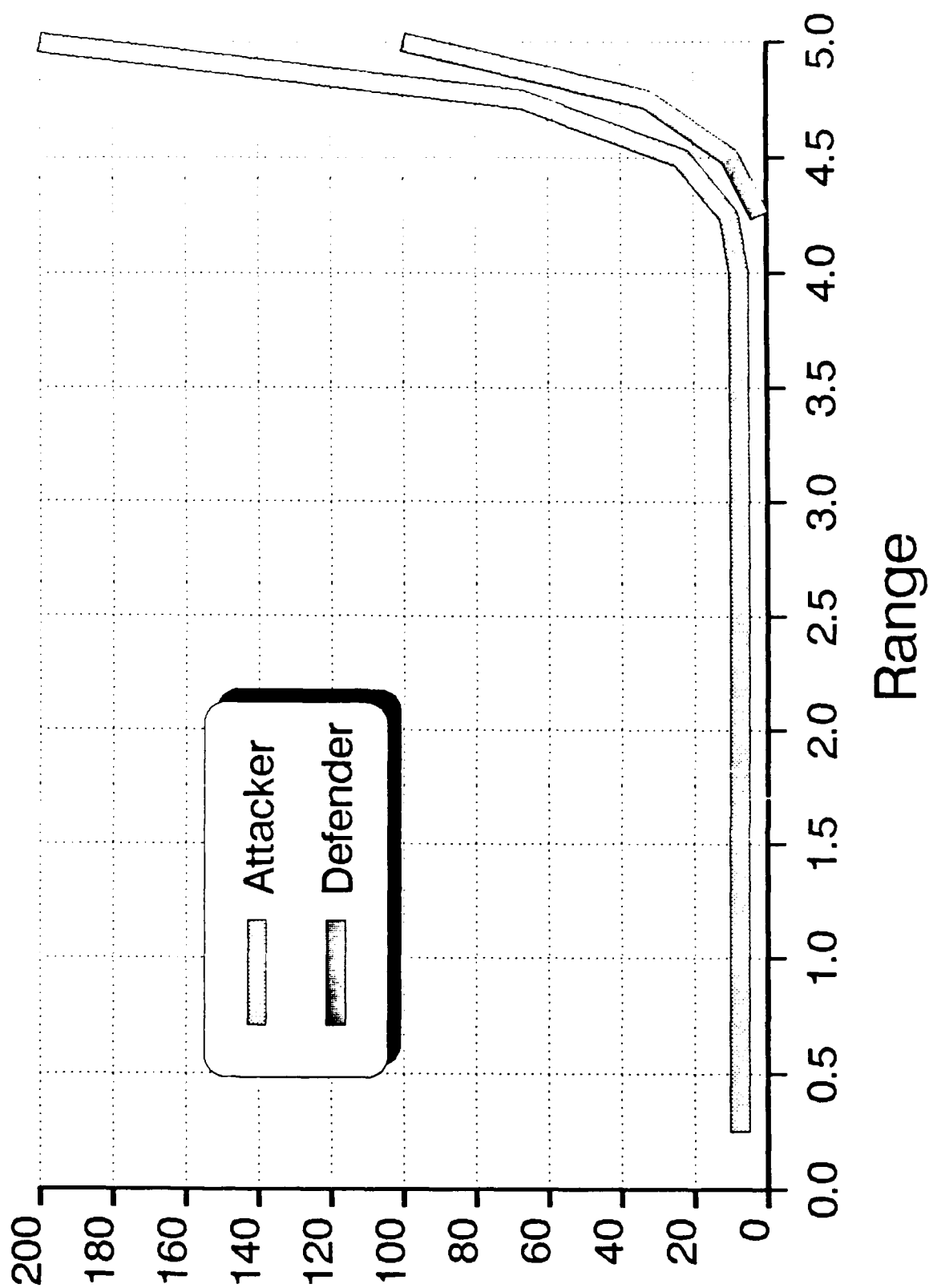
## FORCE STRENGTH

### Fixed Attrition Rates

If we take a fixed attrition rate for both forces (e.g., the value at 2 km), and solve Lanchester's Square Law for a force ratio of 2:1 attackers to defenders (and translate time into range using an advance rate of the attacker), then the force strengths as a function of range (decreasing range equals increasing time--the battle begins at 5 km), then we see that for these force strengths and (fixed) attrition rates, the battle is concluded by the time the attacker has closed to about 4.25 km, and that the attacker is virtually destroyed. This is indeed (by design) a pyrrhic victory. We have ignored the problem of decreasing number of targets to acquire.

# Force Strength

Fixed Attrition Rates



## FORCE STRENGTH

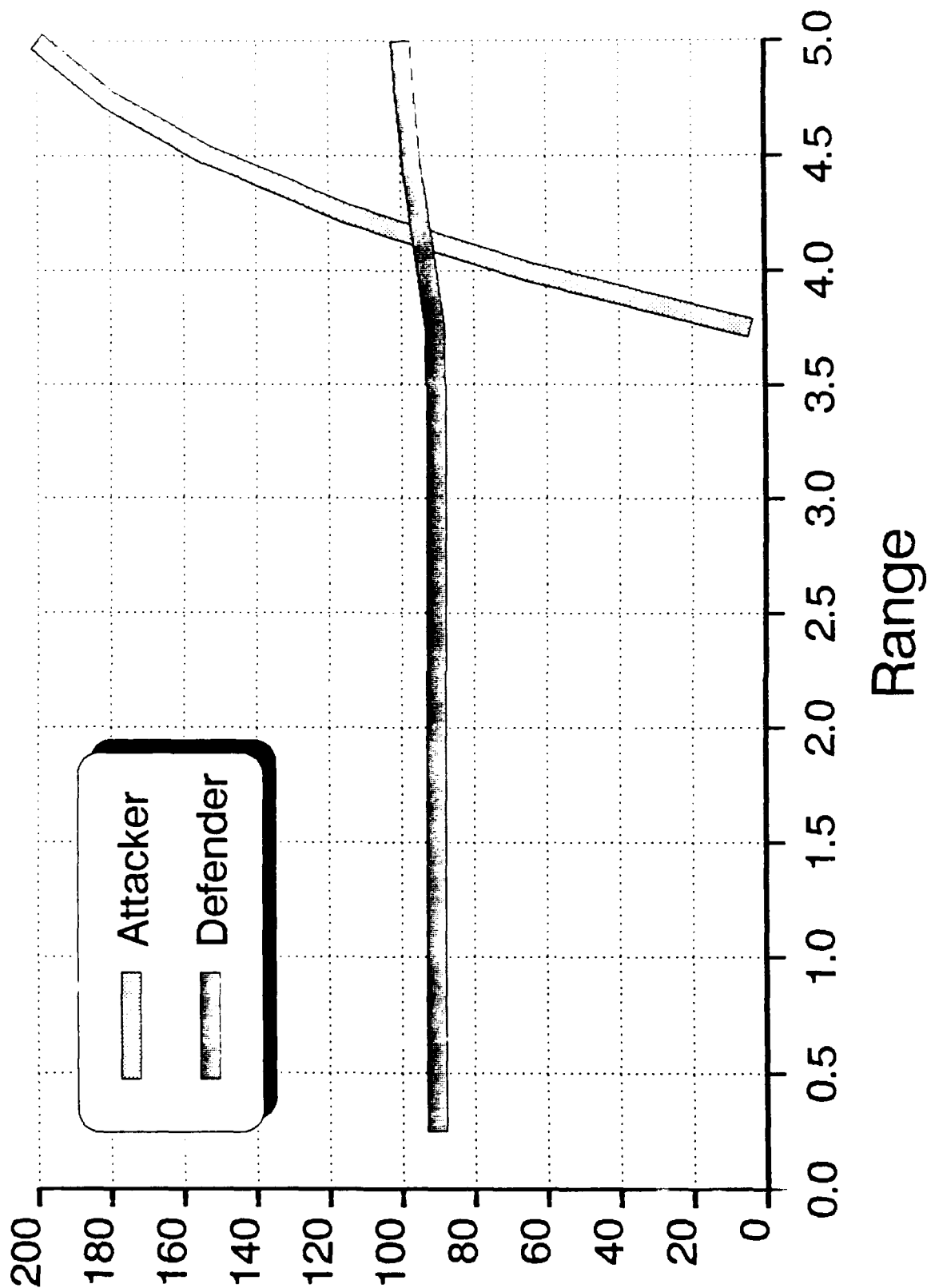
### Variable Attrition Rates

If we now take the variable attrition rates shown earlier and solve Lanchester's Square Law (numerically, since the attrition rates change), we get an entirely different outcome. In this case, the attacker is destroyed by the time he has advanced to about 3.75 km, and the defender takes only about 90%. This is a complete turnaround in results.

The key conclusion to be drawn from this example is that the Lanchester equations are considerably more complicated than would be presumed from the basic assumptions which are necessary to arrive at closed form analytical solutions. Care must be taken in combat modeling and simulation to account for major factors which influence the outcome of battles.

# Force Strength

Variable Attrition Rates



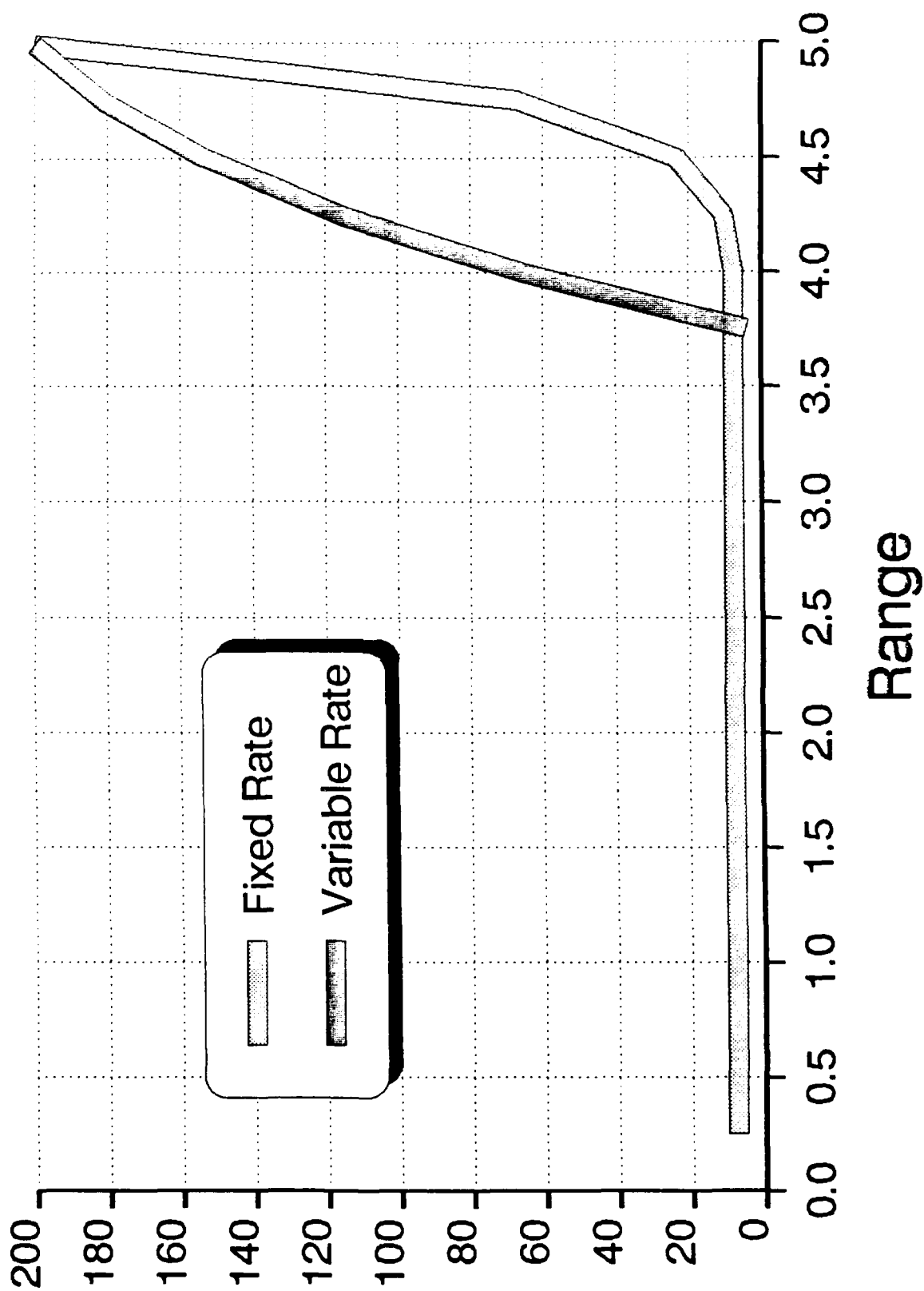
## FORCE STRENGTH

### Attacker

This chart compares the force strength of the attacker versus range for the two cases of fixed and variable attrition rates.

# Force Strength

Attacker



## FORCE STRENGTH

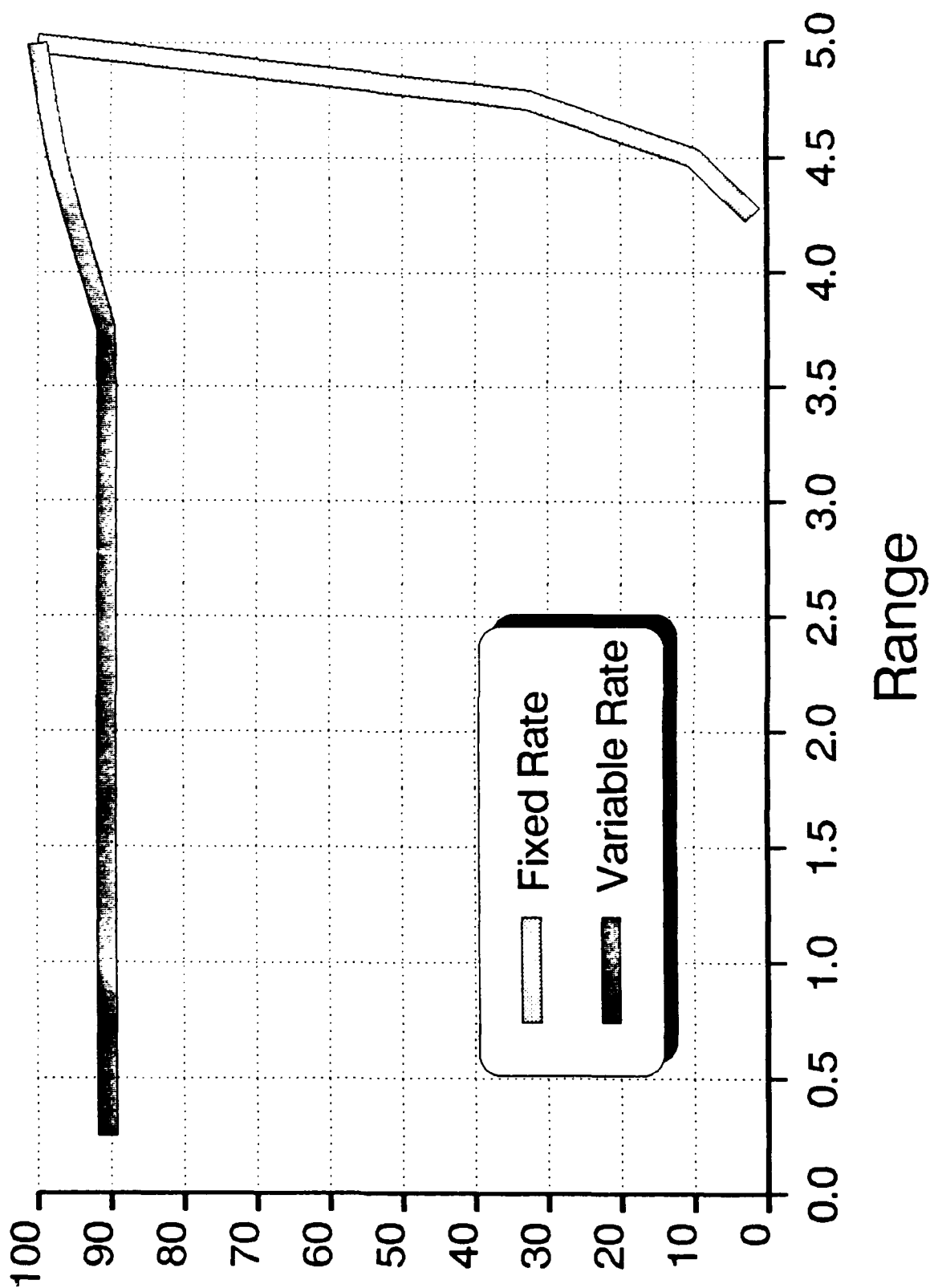
### Defender

This chart compares the force strength of the defender versus range for the two cases of fixed and variable attrition rates. They amply demonstrate the adage of the advantage of the defender over the attacker merely by the introduction of one degree of freedom in the Lanchester equations--range dependence.



# Force Strength

Defender



## **THE TACTICAL WEAPON GUIDANCE AND CONTROL INFORMATION ANALYSIS CENTER (GACIAC)**

GACIAC is a DoD Information Analysis Center operated by IIT Research Institute under the technical sponsorship of the Joint Service Guidance and Control Committee with members for OUSDRE, Army, Navy, Air Force, and DARPA. The U.S. Army Missile Command provides the Contracting Officer's Technical Representative. Its mission is to assist the tactical weapon guidance and control community by encouraging and facilitating the exchange and dissemination of technical data and information for the purpose of effecting coordination of research, exploratory development, and advanced technology demonstrations. To accomplish this, GACIAC's functions are to:

1. Develop a machine-readable bibliographic data base -- currently containing over 36,000 entries;
2. Collect, review, and store pertinent documents in its field of interest -- the library contains over 11,000 reports;
3. Analyze, appraise and summarize information and data on selected subjects;
4. Disseminate information through the GACIAC Bulletin, bibliographies, state-of-art summaries, technology assessments, handbooks, special reports, and conferences;
5. Respond to technical inquiries related to tactical weapon guidance and control; and
6. Provide technical and administrative support to the Joint Service Guidance and Control Committee (JSGCC).

The products and services of GACIAC are available to qualified industrial users through a subscription plan or individual sales. Government personnel are eligible for products and services under block funding provided by the Army, Navy, Air Force and DARPA. A written request on government stationery is required to receive all the products as a government subscriber.

Further information regarding GACIAC services, products, participation plan, or additional copies of this Special Report may be obtained by writing or calling: GACIAC, IIT Research Institute, 10 West 35th Street, Chicago, Illinois 60616-3799, Area Code 312, 567-4519 or 567-4526.

# JSGCC

JOINT SERVICE GUIDANCE AND CONTROL COMMITTEE

